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No. 896.

A DIRECT METHOD OF SPACING RIVETS AND FINDING THE POSITION, ETC., OF STIFFENERS IN PLATE GIRDERS.

By E. SCHMITT, Assoc. M. Am. Soc. C. E.

PRESENTED DECEMBER 5TH, 1900.

WITH DISCUSSION.

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Even if one knows by heart some of the more important formulas of applied structural mechanics, the following axiom will govern their general usefulness:

"Good formulas, however short, are of almost no value in hurried daily practice, where quick and accurate results are expected, unless presented in the shape of tables."

These, again, are serviceable only when in such profusion, and so extended into final results, that, after having been consulted once, no further computations need be performed. They must, also, meet current demands regarding the information they are expected to furnish for all the ordinary problems coming under their headings.

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These, again, are serviceable only when in such profusion, and so extended into final results, that, after having been consulted once, no further computations need be performed. They must, also, meet current demands regarding the information they are expected to furnish for all the ordinary problems coming under their headings.

The writer, and no doubt many others with him, when making structural computations, not having at his disposal labor-saving devices, in the shape of suitable diagrams and tables, and when not wishing to depend on results of rules of thumb, has found the foregoing axiom to be vexatiously true.

Having had to compute plate girders quite frequently, and having found the figuring of rivet spacing, buckling values of webs, location of stiffeners, etc., by formulas, a slow, time-consuming operation, the writer determined to make a short cut for solving such problems, and hopes that the burdens of those who have to compute plate girders will be lessened by the use of the method presented herewith.

It is hoped that the reader will not object to being reminded of the virtues possessed by the "method of graphical statics," which excels greatly the analytic method of solving problems in structural mechanics. It furnishes at once not only particular, but all possible, values and conditions of forces and stresses acting upon structures. Formulas of the analytic method, on the contrary, furnish only one value at a time. Graphical statics reveals to the eye the play of forces and stresses; nothing is hidden; it gives object lessons instead of abstract demonstrations. It possesses, in itself, a complete check of its operations and its results, or it can be made to do so in an easy manner. The analytic method, it is true, gives absolutely correct results, but only in theory, and for the assumed conditions. Such exact fulfilment in assumptions and results cannot be realized in any case, in the working of the forces in the universe or in any structure.

All who have used both methods agree that the graphical is shorter, more comprehensive, and in full accord with assumptions and practical conditions. The results obtained thereby are just as accurate and as true as the assumed conditions on which the computation is based.

Versatility with this method (acquired by constantly using it) is necessary to appreciate fully its perfect qualities in solving the most simple as well as the most intricate problems, statical or elastic, of applied mechanics. It is to be mentioned, however, that all graphical solutions start from analytic formulas, and can never be applied to advantage without first having had a thorough training in the analytic handling of the problems of applied mechanics. When a problem has

to be solved, both methods should be used, each at the proper time, the analytic formulas to be consulted in advice.

No explanation need be given for the manner in which the shear diagram for the vertical shearing stress has been made use of, as will be seen in what follows, for determining the rivet spacing by means of the vertical shear ordinates, directly, although the rivets are acted upon by a horizontal force. The intensity of both these stresses is the same at any point in the web, and since, for the purpose mentioned, the magnitude only of the horizontal shearing stress is of importance, no notice need be taken of its real sense and direction. The position in which the magnitude of this force is found in the shear diagram, is besides a very convenient one for the solution of the problem.

The method to be set forth to determine the rivet pitch and also that point from which on, toward the end of the girders, stiffeners will be required, is graphical, and requires the plotting of the shear curve for the particular loading of the girder, which in any case is a simple performance. For convenience and clearness in carrying out operations, it will be found best to arrange the shear curve immediately above or below the girder to be designed. The depth of the girder may be drawn to any reasonable scale which will allow accurate measurement of the rivet spacing.

Assuming the reader to be familiar with the theory and the designing of plate girders in general, and that he has, for a particular case, decided upon the thickness of the web and the size of the rivet; further, that he has determined the rivet value by means of a rivet table, to be found in any of the mill handbooks (see also the diagram and tables at the end of this paper), we can proceed to refer to the well-known rivet formula:

Let S = shear at any point X in the length of the girder;

h = depth of the girder between the upper and lower pitch lines;

p = the pitch, or spacing, of the rivets to be found;

v = the rivet value;

$p = \frac{v\,h}{S}$; transforming, we have then:

$\frac{h}{p} = \frac{S}{v}$; that is, the depth of the girder is to the rivet pitch, as is the total shear to the rivet value.

Referring now to Fig. 1, let $A\ B = h\ h$ be the span of the girder, and $C\ D$ the shear curve, drawn in reference to the base line $E\ F$. Set off vertically downward at points A and B the depth h of the girder, as Ah, Bh.

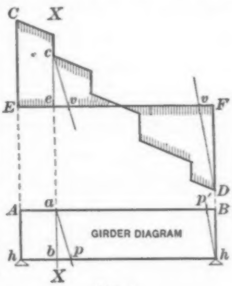


Fig. 1.

Choose any section, as X, and draw a vertical line $c\ e\ a\ b$. The ordinate $c\ e$ will then represent the total shear at X for the panel to the right of e. From the point e in the shear diagram set off, toward the right hand, the rivet value $e\ v$, in the scale of shears. Produce the line $c\ v$. Through the point a, in the girder diagram, draw a parallel line intersecting $h\ h$ in p, then the distance $b\ p$ is equal to the required rivet spacing for the shear at the point X. In the similar triangles $c\ e\ v$ and $a\ b\ p$ we have the proportion:

$$a\,b : p\,b :: c\,e : e\,v,$$

or,

$$h : p :: S : v,$$

and as before,

$$\frac{h}{p} = \frac{S}{v}.$$

To express $b\,p$ in inches, measure the length of the line $b\,p$ in the scale in which the depth h of the girder $A\ B$ has been set off.

We will now proceed to find the position of the stiffeners, or that point s, from which on, toward the abutments, stiffeners will be required.

Referring to Fig. 2, we make use of the shear diagram as before. Set off vertically, in the shear scale, above and below the base line $E\ F$, the buckling, or total shear value w of the web, whatever value the designer has determined to use.

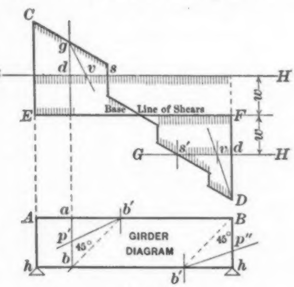


Fig. 2.

The shear stress between the lines EF and $G\,H$ (in the upper part of the diagram) is taken care of by the web. The shear stress beyond the line $G\ H$ will have to be taken care of by stiffeners. Stiffeners will be required therefore, toward the ends beyond the point s, where the line $G\ H$ intersects the shear curve $C\,D$. (See also the lower half of the shear diagram.)

From the theory of the plate girder, it is known that stiffeners, if required at all, must be spaced at distances equal to the depth of the girder. It is also known that at any point in the web, as mentioned before, horizontal and vertical shearing stresses are of the same intensity, and that therefore the spacing of the rivets in the stiffeners can be found in the same manner as for the flange angles; considering, however, that only that part of the shearing stress above the line $G\,H$ (in reference to the base line $E\,F$ and the upper part, Fig. 2), and from the point s on, toward the abutments, has to be taken care of by the stiffeners. The same applies to the lower half of the diagram.

Proceeding now to space off from the point s, distances d, ... etc., equal to the depth of the girder, the ordinate $d\,g$, above the points d, d, represents the surplus shears coming upon the respective stiffeners.

From the stiffener panel-points d set off again the rivet value v, as $d\,v$, produce the line $g\,v$ for any of the stiffeners, and through the point b' in the girder diagram, corresponding to the particular stiffener, draw a normal line to the direction of the line $g\,v$ intersecting the stiffener line $a\,b$ in the point p'. The point b' having been located at a distance $a\,b'$ equal to the depth of the girder, from the point a, by means of the 45° line $b\,b'$.

In the similar triangles $b'\,a\,p'$ and $g\,d\,v$, the same ratio $\frac{h}{p'}, = \frac{S}{v}$ previously found for the flange angles, will obtain, as it should from the statements previously made.

The spacing of the rivets in the stiffeners, thus found, will be, usually, greater than the maximum spacing permissible in rivet work, and should be limited to this accordingly. The size of the stiffener sections may be determined by treating them as columns (with flat ends). Since, however, they are riveted at short intervals along their length, they may be computed for simple compression in ordinary cases, but they should always be applied in the form of angle irons.

The method shown for rivet spacing for particular panels, and for particular shearing stresses, can be reversed in such a manner as to determine the length of the panels in which a certain assumed rivet spacing, for example, 6-in., 4-in. or 3-in., shall take place.

In the girder diagram $A\,h\,B\,h$, Fig. 3, set off, from the point h, the desired rivet spacing, as $h\,p^3$, $h\,p^4$, $h\,p^6$.... From these lines draw lines to the point A, as a pole. Now, in the shear diagram, $C\,E\,F\,D$, set off from E the rivet value v, as $E\,v$. Through the point v draw parallel lines to the lines $A\,p^3$, $A\,p^4$, $A\,p^6$,.... as $v\,p^3$, $v\,p^4$, $v\,p^6$.... intersecting the end shear lines $C\,E$ and $E\,F$ in p^3, p^6.... etc. Through these points, p^3, p^4, p^6, in the shear diagram draw parallel lines to the base line $E\,F$. From the intersection points of these horizontal lines with the shear curve drop verticals to the base line $E\,F$. These ordinates will divide the girder into the desired panels, in each of which the respective assumed rivet spacing will obtain as in Fig. 3.

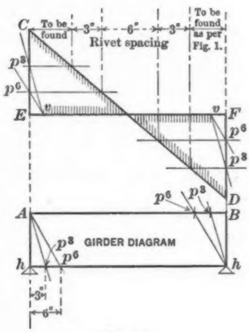


FIG. 3.

In like manner (see Fig. 4) we can find, say for the end of a girder, the rivet value v for a certain pitch by simply setting off the pitch $h\,p$ = say 3 ins. in the girder diagram, and proceeding in the reverse order of the operations carried out in Fig. 1, by drawing the line $A\,p$ and a line $C\,v$ parallel to it through the point C intersecting the base line of the shears in v, then $E\,v$ is the necessary rivet value. Scaling this value and inspecting the rivet table, the thickness of the web and the size of the rivet can be chosen with due regard to practical considerations.

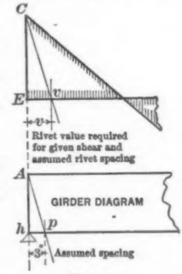


FIG. 4.

The method set forth, so far, gives all the data required for rivet spacing in ordinary practice and theory.

In railroad work, the computation must include, besides the horizontal component due to the main shear, the local vertical effect due to a maximum wheel load. This wheel load is usually assumed to be uniformly distributed over one or two sleeper spaces of 2 to $3\frac{1}{2}$ ft.

As the foregoing forces act at right angles to one another, their resultant must be first ascertained before the proper rivet spacing can be determined.

Let S = main shear, acting horizontally;
h = depth of girder, in feet;
Q = maximum wheel load, acting vertically;
k = the sleeper space, in feet.

Then $\frac{S}{h}$ = shear per lineal foot, due to the main shear,

$\frac{Q}{k}$ = " " " " " " local shear.

The resultant of these two forces, to be taken up by n rivets per foot, will be :

$$r = \sqrt{\left(\frac{S}{h}\right)^2 + \left(\frac{Q}{k}\right)^2}$$

Expressing the sleeper space k in terms of the depth of girder h, by putting $k = \frac{1}{q}\, h$, or $q = \frac{h}{k}$, the above formula can be written

$$r = \frac{\sqrt{S^2 + \left(Q\,\frac{h}{k}\right)^2}}{h}$$

Further, let n = number of rivets per lineal foot;
v = the rivet value;
p = the rivet pitch;

also $n\,p = 1$ (ft.), $p = \frac{1}{n}$ and $n = \frac{1}{p}$, and considering that the condition $n\,v = r = \frac{v}{p}$ must also be fulfilled, we have

$$\frac{v}{p} = \frac{\sqrt{S^2 + \left(Q\,\frac{h}{k}\right)^2}}{h}$$

This formula is, as will be found by comparison, identical with the original rivet formula, presented at the beginning of this paper, and we will use it again as the basis in the determination of the rivet spacing in the present case by evaluating graphically the term under the root, which is the aggregate resultant of all the shearing forces acting upon the n rivets at section X.

This is readily done by forming the "triangle of forces," with the components S and $Q\,\frac{h}{k}$, acting at right angles to one another. The hypothenuse of this triangle will at once represent the term

$$\sqrt{S^2 + \left(Q\frac{h}{k}\right)^2}$$

Referring now to Fig. 5 and section X, let $C\,F$ represent the shear curve for the rolling load, $G\,D$ that for the uniform load, both in reference to the line $E\,F$. Then the ordinate $c\,u$ will be in magnitude equal to S, and will form one of the sides of the triangle of forces we desire to construct. Set off in the shear diagram horizontally from the point E the maximum wheel load Q, as $E\,q$ (in the scale of shears). Similarly, set off in the girder diagram the tie, or sleeper space k (in the scale used for laying off depth of girder), as $h\;K$. Draw the imaginary line $K\,A$ and parallel to it another, $q\,d$, through the point q in the shear diagram, piercing the end shear line in d; then $E\,d$ is equal to the magnitude of the local vertical shear component $Q\frac{h}{k}$, which must be joined at right angles to S.

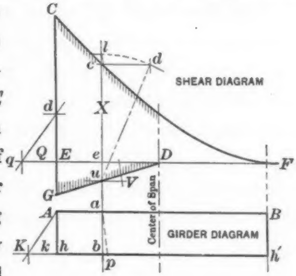


FIG. 5.

In the similar triangles $A\,h\,K$ and $d\,E\,q$, we have—

$$h\,K : A\,h :: E\,q : d\,E,$$

or,

$$d\,E = \frac{E\,q \times A\,h}{h\,K} = Q\frac{h}{k}$$

Now set off, horizontally, from point c in the shear curve $C\,E$, this value $E\,d$, as $c\,d$; draw the imaginary line $u\,d$ (produced in the diagram), then $u\,d$ is equal to the magnitude of the resultant referred to before. Bring this resultant into a vertical position with a compass, using the point u as a center for describing the arc $d\;l$. Set off, further, from the point u, horizontally, the value of the adopted rivet, as $u\;V$; draw the imaginary line $V\,l$, and parallel to it another, through the point a in the girder diagram, piercing the base line of the girder in the point p; then $b\,p$ is equal to the required rivet spacing at the section X, depending on the combined action of the several shearing stresses mentioned.

Practical Points: Web Thickness, Panel Lengths, Etc.—As stated, the web thickness is usually proportioned for the greatest shear and for buckling. For this case no stiffeners are required to assist the web. If, however, the web is calculated for simple shear only, or for

an intermediate condition between buckling and shear, which is also frequently done, then the method just shown for locating the stiffeners is to be used.

The rivet value of a rivet depends on the shearing capacity of the particular rivet and the web thickness in which it bears, etc., and the lesser of the two values, usually that for bearing, must be adopted to determine the pitch. If this pitch should be found to be less than the minimum spacing permissible, a larger rivet or a heavier web must be resorted to. To obtain, under certain circumstances, a sufficient rivet value as regards bearing, without changing either the diameter of the rivet, which may be a maximum, or the spacing, which may be a minimum, already, a deeper web must be selected.

This depth can be determined at once by means of the diagram, Fig. 4. In the case just stated the known quantities are: The total shear, the rivet value, consequently the direction of the line $C\ v$, to which draw a parallel line $p\ A$ through the point p, intersecting the abutment line of the girder in the point A; then $A\ h$ is equal to the depth of the girder which is sought.

The minimum spacings for rivets are: $2\frac{1}{4}$ ins. for a $\frac{3}{4}$-in. rivet, $2\frac{5}{8}$ ins. for a $\frac{7}{8}$-in., and 3 ins. for a 1-in. rivet.

The panels in which the rivet spacing changes, or should change, are, in the case of concentrated loading, given directly. For a uniform load, or a uniform and concentrated loading combined, suitable panel lengths may be assumed, mostly equal to the depth of the girder. The designer, with a little practice, however, will be able to settle this point without difficulty. The panel lengths also can be determined in such a manner as to obtain in them certain rivet spacings, as illustrated in Fig. 3.

Stiffeners are always provided over the supports, or where concentrated loading occurs, even if the web has been designed to take the greatest shearing stresses for buckling.

As this shearing stress is greatest at the ends, it is taken usually to determine the web thickness. Since, however, stiffeners are always provided at the ends, the writer would recommend, for the purpose of selecting a suitable web thickness, the use of the shear which obtains at a distance equal to the depth of the girder from the ends, or even at a still greater distance.

If the pitch for any panel point has been determined, it is con-

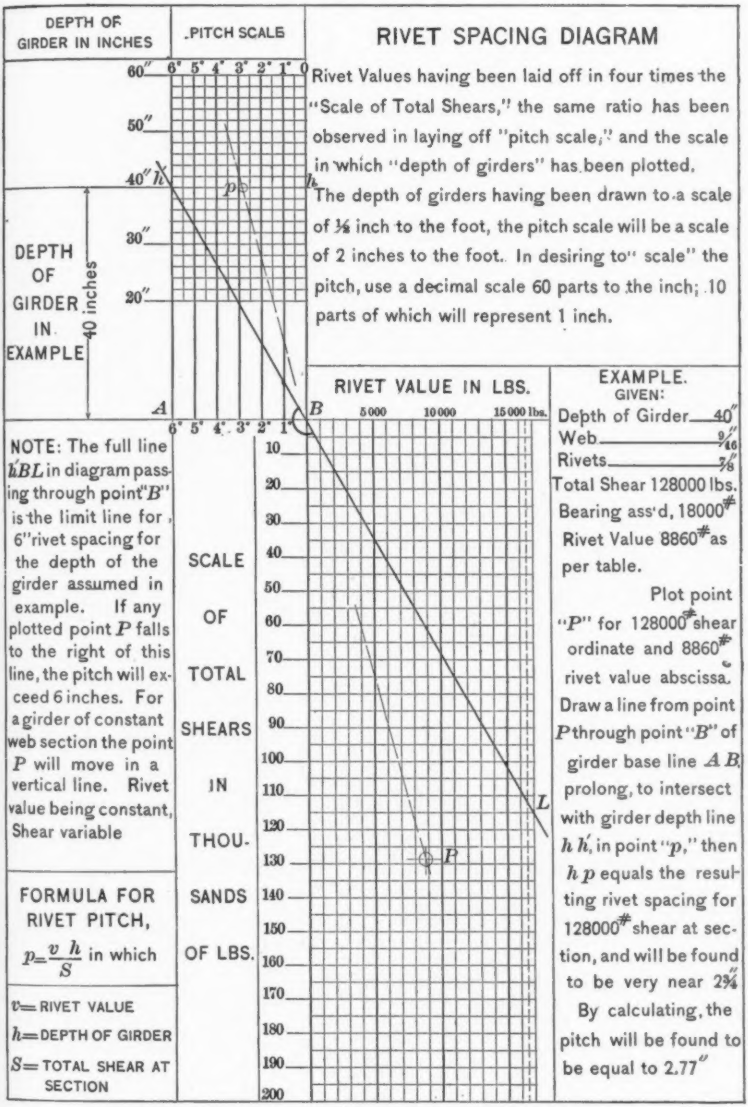

DEPTH OF GIRDER IN INCHES
.PITCH SCALE
RIVET SPACING DIAGRAM
Rivet Values having been laid off in four times the "Scale of Total Shears," the same ratio has been observed in laying off "pitch scale," and the scale in which "depth of girders" has been plotted.
The depth of girders having been drawn to a scale of 1/8 inch to the foot, the pitch scale will be a scale of 2 inches to the foot. In desiring to "scale" the pitch, use a decimal scale 60 parts to the inch; 10 parts of which will represent 1 inch.
60″
50″
40″
30″
20″
6″ 5″ 4″ 3″ 2″ 1″ 0
h
h′
p
A
B
L
P
DEPTH OF GIRDER IN EXAMPLE
40 inches
RIVET VALUE IN LBS.
5000
10000
15000 lbs.
EXAMPLE.
GIVEN:
Depth of Girder 40″
Web 9/16″
Rivets 7/8″
Total Shear 128000 lbs.
Bearing ass'd, 18000#
Rivet Value 8860# as per table.
Plot point "P" for 128000# shear ordinate and 8860# rivet value abscissa. Draw a line from point P through point "B" of girder base line A B, prolong, to intersect with girder depth line h h′, in point "p," then h p equals the resulting rivet spacing for 128000# shear at section, and will be found to be very near 2¾″
By calculating, the pitch will be found to be equal to 2.77″
NOTE: The full line h′BL in diagram passing through point "B" is the limit line for 6″ rivet spacing for the depth of the girder assumed in example. If any plotted point P falls to the right of this line, the pitch will exceed 6 inches. For a girder of constant web section the point P will move in a vertical line. Rivet value being constant, Shear variable
SCALE OF TOTAL SHEARS IN THOUSANDS OF LBS.
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
FORMULA FOR RIVET PITCH,
$p = \frac{v\,h}{S}$ in which
v = RIVET VALUE
h = DEPTH OF GIRDER
S = TOTAL SHEAR AT SECTION

FIG. 6.

tinued to the next panel point toward the center, for practical reasons. Theoretically, and as may be seen from the shear diagram and from the method by which the pitch is obtained, the distance from rivet to rivet changes gradually, or abruptly, as does the shearing stress. To realize such constantly varying spacing of rivets in girders would be very impractical and expensive unless performed by an automatically arranged punching machine. In any case, the rivets are never spaced more than 6 ins. apart, and, in case of girders carrying floors, the spacing is recommended not to exceed 4 ins.

Spacing-panels for rivets should always contain an even number of spaces. The usual spacing for rivets should be preferably given in full inches or full and half inches. Quarter-inch fractions, however, are frequently used.

The foregoing method of finding the rivet spacing in web members of plate girders has been compacted in the diagrams, Figs. 6 and 7. Auxiliary tables (Tables Nos. 1, 2 and 3) have been added to give all the necessary data to design the web plates of girders without having to inspect books or tables.

If the analytic method is used, the diagrams and tables will be of advantage in computing shear stresses. The necessary explanations of how to use the diagrams are noted at the proper places. An example is given for finding the rivet pitch for a given total shear of 128 000 lbs. for a $\frac{7}{8}$-in. rivet in a $\frac{9}{16}$-in. web, and for 18 000 lbs. bearing per square inch of rivet diameter.

It may be noted that the points P (see lower part of rivet spacing diagram), for a particular girder, with constant web section, will all be located in a vertical line, since the rivet value remains constant; the shearing stresses only varying along the length of the girder. The shear diagram, for finding the shear per square inch of web area of a given or assumed web, can be used in the reverse manner, that is, to find the necessary total web section for a given or assumed unit shear per square inch and a given total shear.

The manner of doing this is given indirectly in the explanation of how to use the shear diagram. By entering the table of web areas, the web thickness will be found with due regard to the assumed depth of the girder.

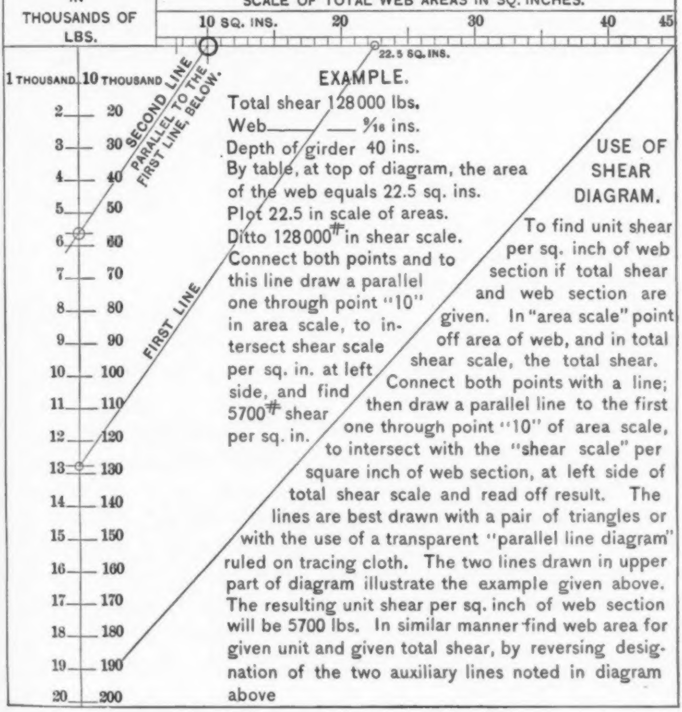
Allusion will have to be made to the term "depth of girder."

IN SHEAR SCALE BELOW ON

THIS SIDE	THIS SIDE
FIND RESULTING OR ASSUMED	
SHEAR PER SQ. IN.	TOTAL SHEAR
OF	ON
WEB SECTION IN THOUSANDS OF LBS.	

THICKNESS OF WEB IN INCHES		AREAS OF WEBS FOR DEPTHS OF										
		24	28	30	32	36	40	42	48	50	54	60″
	$\frac{5}{16}$	7.5	8.75	9.37	10.0	11.25	12.5	13.12	15.0	15.6	16.87	18.75
$\frac{3}{8}$		9.0	10.5	11.25	12.0	13.5	15.0	15.75	18.0	18.75	20.25	22.5
	$\frac{7}{16}$	10.5	12.25	13.12	14.0	15.75	17.5	18.38	21.0	21.87	23.62	26.25
$\frac{1}{2}$		12.0	14.0	15.0	16.0	18.0	20.0	21.0	24.0	25.0	27.0	30.0
	$\frac{9}{16}$	13.5	15.75	16.87	18.0	19.75	22.5	23.62	27.0	28.12	30.37	33.75
$\frac{5}{8}$		15.0	17.5	18.75	20.0	22.5	25.0	26.24	30.0	31.25	33.75	37.5
	$\frac{11}{16}$	16.5	19.25	20.62	22.0	24.75	27.5	28.86	33.0	34.38	37.12	41.25
$\frac{3}{4}$		18.0	21.0	22.5	24.0	27.0	30.0	31.5	36.0	37.5	40.5	45.0

DIAGRAM FOR SHEAR PER SQ. IN. OF WEB AREA
FOR TOTAL SHEARS UP TO 200000 LBS. AND TOTAL WEB AREAS OF 45 SQ. INS.



EXAMPLE.

Total shear 128000 lbs.
Web——— ——— 9/16 ins.
Depth of girder 40 ins.
By table, at top of diagram, the area of the web equals 22.5 sq. ins.
Plot 22.5 in scale of areas.
Ditto 128000# in shear scale.
Connect both points and to this line draw a parallel one through point "10" in area scale, to intersect shear scale per sq. in. at left side, and find 5700# shear per sq. in.

USE OF SHEAR DIAGRAM.

To find unit shear per sq. inch of web section if total shear and web section are given. In "area scale" point off area of web, and in total shear scale, the total shear. Connect both points with a line; then draw a parallel line to the first one through point "10" of area scale, to intersect with the "shear scale" per square inch of web section, at left side of total shear scale and read off result. The lines are best drawn with a pair of triangles or with the use of a transparent "parallel line diagram" ruled on tracing cloth. The two lines drawn in upper part of diagram illustrate the example given above. The resulting unit shear per sq. inch of web section will be 5700 lbs. In similar manner find web area for given unit and given total shear, by reversing designation of the two auxiliary lines noted in diagram above

Fig. 7.

This term has several distinct and different meanings, relative and specific, all of which must be fully understood and correctly applied.

Generally, and as ordinarily understood, the depth of the girder is that fractional part of the span which constitutes its height, in common parlance, and in most cases is settled upon first and definitely after a thorough consideration of conditions, whether restrictions as to depth and width have to be observed, or whether the designer is at liberty to choose these, according to his best judgment. After the depth, etc., has been assumed in this general way, the other specific depths to be mentioned are derived therefrom as the design progresses, commencing with the flanges.

In ordinary building practice, for instance, if no restrictions are placed on the depth of girders, it is found (the result being in inches) by multiplying the span of the girder (expressed in feet) by six-tenths, and adding to the product the constant amount, 3 ins. The rate of depth of girder to length of span is, in this instance, one-twentieth plus the constant.

In heavier work the multiplier is eight-tenths, equal to a one-fifteenth rate. In bridge work the proportion of depth to span is increased to from one-twelfth to one-eighth, corresponding to multipliers of 1 and 1.5, respectively.

Having explained the first meaning of the term "depth of girder," the other meanings will now be referred to.

In determining the rivet spacing of the rivets connecting the web plate to the flange angles the depth of the girder is the vertical distance between the horizontal rivet lines located in the vertical legs of the top and bottom chord angles.

If the buckling value of the web plate is concerned, the depth of the girder is the clear distance between the inner edges of the vertical legs of the same angles.

The fourth subdivisable meaning of this term is involved when flange stresses are computed. If these are determined by the ordinary approximate method, the depth of the girder is taken from out to out of flange angles, if cover plates have to be provided to make up the required flange area. If none are needed, the depth is equal to the vertical distance between the centers of gravity of each pair of upper and lower flange angles.

If the exact method of moment of resistance, or, more correctly,

that of the section modulus, is selected to determine the cross-section of a plate girder, then we have to introduce into the computation for the depth (no flange plates) the distance "out to out" of angles, as before. With flange plates, take the depth over all the plates; the depth will change wherever a plate is dropped off.

There exists a sixth condition regarding this term, frequently overlooked by younger designers, and of which no mention is made in treatises on applied mechanics, in that the depth "out to out" of flange angles, as a rule, is greater by ½ in. than the depth of the web plate over all. This allowance of ¼ in., at top and bottom, is made to avoid the projection of any part of the web beyond the flange angles. This allowance, therefore, does away with extra work, either shearing the web beforehand, or chipping it afterward, should the angles and web have been required to be flush.

If universal mill plates are specified for the web, some engineers do not demand this allowance to be made. Others insist on having it observed in every instance, as do most all of the mills in their shop practice. For special purposes, however, it may be required that the web shall project a certain amount beyond the backs of the angles; this is the exception to the rule.

Splicing in webs or angles in plate girders, in sizes ordinarily called for in architectural building practice, is not necessary, as universal mill plates for the web are rolled in dimensions from ⅜ in. by 70 ft., up to ¾ in. by 65 ft., and angles on the average run up to a length of from 60 to 75 ft.

In conclusion, the attention of the reader is directed to two noteworthy papers on plate girders. The first, by J. C. Bland, M. Am. Soc. C. E., in the 1887 handbook of the Pottsville Iron and Steel Company. The second*, somewhat more extended in its scope, by Isami Hiroi, C. E., Professor of Civil Engineering in Sapporo College.

To fully complete this paper, and illustrate the graphical method for finding the rivet spacing in plate girders, the writer has chosen for exemplification the problem worked out by Mr. Hiroi, in his booklet, and in which, as regards the determination of the rivet pitch, he has used largely the analytic method of computation. In Fig. 8 will be found all the data requisite to a full understanding of the problem and the steps to be taken to solve it, and to form an opinion as to the

* Van Nostrand's Science Series, first edition, 1888; second, 1893, revised.

comprehensiveness and simplicity of the writer's method. The attention of teachers of structural mechanics is invited to the manner in which the polygon of maximum shear for the rolling or moving loads, has been formed, as fully explained in the "Programme of Example" accompanying the diagram, for determining whether it is not simpler and more readily executed than by the usual method. The difference in rivet spacing, due to assuming one or the other of the meanings of the term "depth of girder" has also been alluded to. It will be seen thereby how easily the rivet spacing can be adjusted to conform to any depth of girder chosen with this method, if otherwise the conditions of the external forces remain the same.

*Programme of Example.**—Single-track standard-gauge deck-girder, 50-ft. span between centers of bed plates, divided into 10 panels, 5 ft. each, for maximum stresses by bending and shears. Depth of girder, "out to out" of angles, 4 ft. 6¼ ins. Depth assumed in computation for stresses, in Hiroi's book* and in this example, 4 ft. 6 ins. = 54 ins. Dead load, 490 lbs. per lineal foot. Rolling load, as per train diagram at top of Fig. 8. Maximum wheel load, 12 000 lbs., and, for local shear computation, assumed to be distributed over a sleeper space of 2 ft. End shear due to dead load, one-fourth of 490 x 50 x 2 = 12 300 lbs. Web thickness, ⅜ in.; rivets, ⅞ in. diameter. For bearing unit of 14 400 lbs., in ⅜-in. web, rivet value 4 700 lbs.; shear unit for web, 6 400 lbs. Flange angles, 3½ ins. x 5 ins. x ½ in; true depth for rivet spacing, 50 ins. (not adopted in this example).

Construct maximum main shear diagram (curve) for rolling load, $E\,F\,C$, as noted. Construct dead-load shear line, $G\,D$, in reference to $E\,F$. Construct local, vertical, shear-component, due to maximum wheel-load (see left lower corner of girder diagram). Construct total maximum shear curve, $M\,L$, as per Fig. 5, up to center of span. Construct girder diagram, using ½-in. scale for the depth. Produce the flange-rivet bevel-lines $V\,l$, $V_a\,l_a$, $V_b\,l_b$, etc., in shear diagram, for each panel point, and draw the rivet-spacing bevel-lines $A\,p$, $a\,p_a$, etc., in girder diagram, and find the rivet spacings $b\,p$, $b_a\,p_a$, etc. Set off, parallel to $G\,D$, the web value $u\,u'$ in shear diagram, computed at eight-tenths of buckling value in Table No. 1, for a ⅜ x 47-in. web (54 ins. — 2 x 3½ ins.) (see also page 63, Hiroi). Construct, as per Fig. 2,

* See "Plate-Girder Construction," by Isami Hiroi, C. E., Van Nostrand's Science Series, No. 95, second edition, 1893, pages 32, 38, 39, 62, 63, 79, 81, 83, etc.

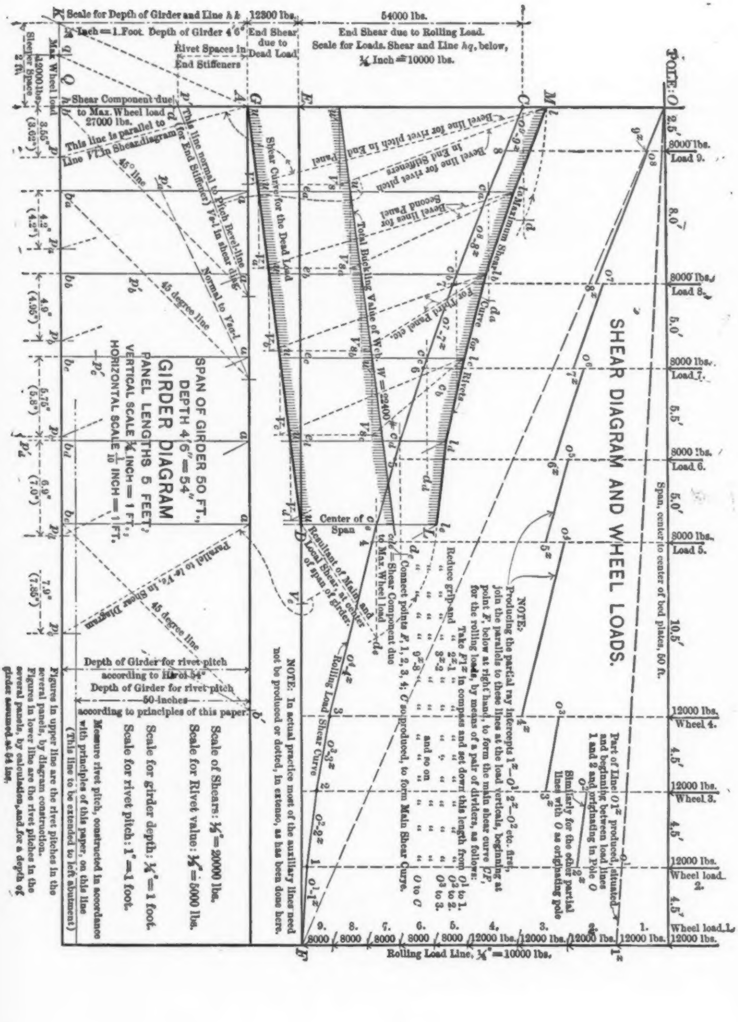

SHEAR DIAGRAM AND WHEEL LOADS.
GIRDER DIAGRAM
SPAN OF GIRDER 50 FT.,
DEPTH 4′6″=54″
PANEL LENGTHS 5 FEET;
Rolling Load Line, ¼″=10000 lbs.
End Shear due to Rolling Load.
54000 lbs.
12300 lbs.
Scale of Shears: ¼″=2000 lbs.
Scale for Rivet value: ¼″=5000 lbs.
Scale for girder depth: ¼″=1 foot.
Scale for rivet pitch: 1″=1 foot.

FIG. 8.

the stiffener-rivet bevel-lines $Vs\ l$, $Vs_a\ l_a$, etc., in shear diagram, and produce the corresponding normal lines to these, in girder diagram, giving the rivet spacing-points in stiffeners, as p', $p'a$, etc. The rivet values, $u\,V$, $u'\,V_s$, $u\,V_a$, $u'\,V_{sa}$, etc., having been laid off at four times the actual rivet value, of 4 700 lbs. (to obtain accuracy), rivet spacings must be measured in a scale having a four times larger denomination than the scale in which the depth of the girder was laid off.

TABLE No. 1.—Total Buckling Values of Webs, $w = \dfrac{10\,000\ d\ t}{1 + \dfrac{d^2}{3\,000\ t^2}}$

Thickness of web, in inches.	Depth of Web, in Inches.											
	24	28	30	32	36	40	42	48	50	54	60	
5/16	25 300	23 750	23 100	22 300	19 700	19 200	18 650	17 200	16 500	15 350	13 850	lbs.
3/8	38 600	36 500	36 100	34 700	33 100	31 400	30 400	27 800	27 350	25 700	21 300	"
7/16	52 500	51 750	51 500	50 750	47 600	47 000	45 300	42 000	40 750	39 200	36 000	"
1/2	68 250	68 500	68 250	67 500	66 000	63 800	63 400	58 800	58 000	53 800	51 000	"
9/16	85 500	85 600	85 250	85 400	83 750	83 200	82 800	77 250	74 750	74 750	70 500	"
5/8	103 000	103 800	106 000	106 700	107 000	106 700	104 500	102 000	99 500	97 000	92 400	"
11/16	105 500	124 750	126 200	128 500	130 050	130 000	129 500	126 500	125 500	121 500	117 000	"
3/4	136 000	142 750	147 000	149 500	152 750	154 000	157 800	152 000	151 000	148 500	143 500	"

TABLE No. 2.—Total Buckling Values of Webs, $w = \dfrac{13\,000\ d\ t}{1 + \dfrac{d^2}{3\,000\ t^2}}$

Thickness of web, in inches.	Depth of Web, in Inches.											
	24	28	30	32	36	40	42	48	50	54	60	
5/16	32 800	30 600	30 000	29 000	25 600	25 000	24 300	22 200	21 500	20 250	18 000	lbs.
3/8	49 500	47 500	47 000	45 100	43 100	40 750	39 500	36 200	35 600	33 500	27 700	"
7/16	68 350	67 300	67 000	66 000	62 000	61 250	58 800	54 750	51 300	51 000	46 800	"
1/2	88 750	89 000	88 650	87 900	85 800	83 000	82 500	76 600	75 500	70 000	61 250	"
9/16	111 000	112 250	112 900	112 000	111 000	109 500	107 500	102 500	101 200	97 250	91 800	"
5/8	132 000	135 000	138 000	138 500	139 000	137 500	136 000	130 250	130 000	126 000	120 000	"
11/16	157 000	162 000	164 700	167 000	169 500	169 000	168 500	164 500	163 000	158 500	152 000	"
3/4	176 750	186 500	191 000	194 000	198 500	200 000	205 000	197 500	196 500	193 000	186 500	"

TABLE No. 3.—SHEARING AND BEARING VALUES OF RIVETS.

Diameter and area of rivet.		Rivet Values.		Thickness of Web or Plate, in Inches.							
		Single shear.	Shear unit = ½ bea'r.	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
				Rivet Values for Bearing.							
$\frac{3}{4}$	in	3 310	7 500	3 520	4 220	4 920	5 630	6 330			
$\frac{7}{8}$	deci-	4 510	and	4 100	4 920	5 740	6 560	7 380	8 200	9 030	
1	mals.	5 890	15 000	......	5 620	6 560	7 500	8 440	9 380	10 310	11 250
$\frac{3}{4}$	0.75	3 980	9 000	4 220	5 060	5 900	6 760	7 600			
$\frac{7}{8}$	0.875	5 400	and	4 920	5 900	6 880	7 880	8 860	9 840	10 820	
1	1.00	7 070	18 000	5 610	6 750	7 880	9 000	10 500	11 220	12 400	13 450
$\frac{3}{4}$	area	4 420	10 000	4 690	5 630	6 570	7 500	8 440			
$\frac{7}{8}$	in	6 010	and	5 470	6 570	7 600	8 750	9 840	10 840	12 300	
1	sq.in.	7 850	20 000	......	7 500	8 750	10 000	11 250	12 500	13 750	
$\frac{3}{4}$	0.4418	4 860	11 000	5 160	6 190	7 220	8 250	9 280			
$\frac{7}{8}$	0.6013	6 610	and	6 020	7 220	8 430	9 630	10 840	12 040	13 240	
1	0.7850	8 640	22 000	......	8 250	9 630	11 000	12 380	13 750	15 130	16 500

NOTE.—Within the extent of the table: Connections with double-shear rivets are governed by the bearing values given; connections with single-shear rivets are governed by the single-shear values given.

DISCUSSION

Mr. Schaub. J. W. Schaub, M. Am. Soc. C. E. (by letter).—The writer wishes to call attention to the author's method of proportioning the intermediate stiffeners to carry the shear on the web of a girder. This is the usual method of proportioning for stiffeners, but it can be shown that, if, in some way, the load can be put into the web, no stiffeners whatever will be required, except, of course, the end stiffeners. This is contrary to all accepted theories of web stresses, but a little experiment can be made to demonstrate it.

The following experiment was made by the writer* before the Western Society of Engineers, October 10th, 1900:

Make a plate-girder model of drawing paper, about 15 ins. long and 3 ins. deep, with two flange angles at the top and bottom, and let the web of the girder project below the bottom flange angles, so that the loads can be suspended by means of hangers attached to the projecting web. Now support the girder at the ends and support the top flange against crippling sideways. It will be found that loads which can be suspended from the hangers with perfect safety cannot be placed on top of the girder without buckling the web. Moreover, the web will stiffen up at once, and it will be found that, although the web is already carrying the suspended loads, additional loads can now be placed on top of the girder and the web will not buckle. It follows, then, that after the load is put into the web, the web is actually stiffer than it was before; so that stiffeners need only be provided to get the load into the web, and not to carry the maximum shear at any point. Would it not be more rational, then, to proportion the intermediate stiffeners to carry only the maximum concentrated load at any point, rather than the maximum shear?

With this experiment in view, the writer has specified, in a specification just issued for a western railroad, as follows:

"Intermediate stiffeners shall be provided for, as struts carrying the maximum concentrated load at any point, including impact, whenever the clear width between flanges exceeds 50 times the thickness of the web, and they shall be spaced at intervals of about the depth of the girder. End stiffeners shall be provided to carry the maximum end shear, including impact."

This method is certainly very simple. For through plate-girders provision need be made for intermediate stiffeners only to attach the floor system to the web, and between the floor beams, or panel points, no intermediate stiffeners are required, no matter how long the panels are, or how thin the web may be; whereas, by the usual method of

* "Proposed Specifications for Steel Railroad Bridges," *Journal*, Western Society of Engineers, October, 1900.

proportioning, intermediate stiffeners are provided for through plate-girders, the same as for deck plate-girders. This leads frequently to some curious results. The writer has in mind a specification for a prominent southern road which required intermediate stiffeners, by a so-called stiffener formula, to be spaced at 2-ft. centers near the ends, and a number of through plate-girders were built accordingly, and are probably still being built according to this specification. Mr. Schaub.

Bernt Berger, Assoc. M. Am. Soc. C. E.—This paper is a valuable exposition of a graphical method of finding the exact spacing of rivets in flanges of plate girders under assumed loads and stresses. It is well worth careful study as it gives a clear insight into the variations in values and conditions of forces and stresses. The speaker has great faith and confidence in graphic statics, and uses that method quite extensively, but must differ with the author on the question of the applicability of the graphical method to the ordinary, every-day work of designing plate girders. The graphical method can at times be used to advantage in computations of railroad bridges for engine diagrams; but, for most of such cases, and especially for plate girders, the speaker has found, in his own work, that the analytical method, applied with the aid of the well-known moment diagrams, is much shorter and easier, especially when a slide rule is also used. For plate-girder bridges, as in the present case, it is, of course, necessary to find the end shear in the first place, and the shear at a very few intermediate points besides. For practical reasons, the rivet spacing in the flanges is constant for certain distances, and for shorter plate girders, therefore, all that is needed is the end shear and the shear at the quarter point. The 6-in. limit of rivet pitch will cover the shear in the middle half, especially when the rule about closer pitch in the flange which carries the track is also taken into account. For longer girders, up to 80 or 100 ft., the shear should be found at a few more points; but, with the aid of a moment diagram, this is readily and quickly done, more so, the speaker has always found, than by any graphical method. Mr. Berger.

The thickness of the web is determined by the required section of metal for shear and the necessary number of rivets. Shallow girders must have a thicker web, deep girders may have a thinner web. Each end stiffener must be able to take all the end shear in compression, but not necessarily as a column of length equal to the depth of the girder, as the web through the rivets distributes the shear over the end stiffener through its whole depth. The intermediate stiffeners cannot, according to the speaker's views, act as columns for the distribution of the shear over the web, as most of that shear is already in the web; but they can so act under concentrated loads, which are only part of the shear, as when engine loading for railroad bridges is

Mr. Berger. considered. Thus, for girders which carry the track direct on one or the other of the flanges, the service which such intermediate stiffeners perform, consists then principally in holding the web in position—preventing it from buckling. But where, for instance, the stringers of a bridge rest on top of the floor beams, the stiffeners on the floor-beam web must of course serve to distribute the concentrated load over the web of the floor beam, and must have sufficient area for compression.

As to the axiom laid down by the author in the beginning of the paper, the speaker begs to say that, generally speaking, he thinks formulas little worth the effort of remembering. The thorough understanding of principles and methods, and their application, is of far greater importance, and to this end the paper is of great help.

Mr. Skinner. F. W. SKINNER, M. Am. Soc. C. E.—A particularly interesting, and perhaps extreme, case of the application of the web stiffener system to a plate girder, and one which the speaker believes is not widely known, came under his observation recently.

In Cincinnati, Ohio, there is a plate-girder span of 213 ft.—the longest of this kind of which the speaker has ever heard. The bridge is formed of two main girders, each 16 ft. deep, which carry a 30-ft. roadway and two 10-ft. sidewalks. The thickness of the web could not be ascertained, but is not more than $\frac{3}{8}$ in., and, possibly, is not more than $\frac{3}{16}$ in. The structure has been in use more than thirty years, and the iron is badly corroded. The bottom chord is made with two small angles and several 42-in. horizontal flange plates. The top-chord is a three-web box section 42 ins. wide. The web is divided into panels of about 10 ft., and in each panel there are two transverse stiffener plates, each 26 ins. wide, and each formed with a flange angle all around.

Calculations may be possible as to how much that web, 16 ft. high and $\frac{5}{16}$ in. thick, would have stood without the stiffeners, and also how much it would stand with the stiffeners.

One peculiar feature was the way in which each panel of the web was built up of five pieces, 8 ft. high, butt jointed, and spliced with two cover plates.

Another peculiar feature of the bridge was the extreme generosity of the bearings on the abutments, each of which consisted of two very thick cast-iron plates about 7 ft. long; the upper one sliding between longitudinal flanges on the edges of the lower plate.

After many years' service, two intermediate steel trestle bents, each about 50 ft. from the center of the span, were put in to reduce the vibrations.

The whole construction is a curiosity, if not a monstrosity, but is still in active service, and is remarkable for its dimensions.

H. A. La Chicotte, M. Am. Soc. C. E.—The author has suggested a graphical method of spacing rivets in plate girders, which is really a graphical solution of a well-known formula: Mr. LaChicotte.

$$p = \frac{v\,h}{s},$$

in which p = the pitch;

v = the rivet value;

h = the depth of girder between rivet lines;

and s = the shear, assumed as more or less uniformly distributed over the section.

Fig. 9 will indicate clearly the derivation of the formula by the method of moments.

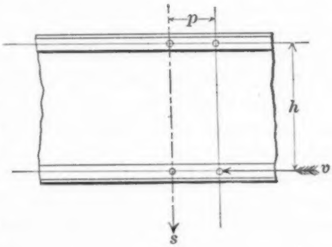


Fig. 9.

According to the common theory of the plate girder, the lines of stress in the web act at an angle of 45° with the vertical, so that the stress induced in the flanges by the web must, in a length of flange equal to the depth of the girder, equal the shear. In other words, there must be enough rivets in either flange, within a length equal to the depth of the girder, to equal the shear.

If, in addition to the horizontal stress in the flanges, the rivets are required to transmit to the web a local load, Q, more or less distributed and acting at right angles to the flange stress, the rivets must be spaced to resist the resultant of the two stresses.

If Q be distributed over a distance k (less than h), the diagonal lines of stress distribute it over an equal distance in the vertical section of the web, and, in order to distribute it with equal intensity over the whole section, Q must be first divided by k to find the unit intensity, and then multiplied by h, or, $\frac{Q\,h}{k}$. The formula for pitch now becomes:

$$p = \frac{v\,h}{\sqrt{s^2 + \left(\frac{Q\,h}{k}\right)^2}}.$$

If the load Q be distributed over a distance equal to h,

$$p = \frac{v\,h}{\sqrt{s^2 + Q^2}}.$$

The application of the formula $p = \frac{v\,h}{s}$ is very simple, after the shear has been computed at the requisite number of points; and, knowing the law of change in the shear, the desired results are quite as quickly obtained by the analytical as by the graphical method. For any particular girder, s being the only variable, the values of p, for

Mr. LaChicotte. any number of points corresponding to known values of s, may all be read from a slide rule by a single setting.

If the stresses in the girders are found graphically, the author's method of spacing rivets is a valuable adjunct, but if these stresses are determined analytically, the time and labor of constructing the diagrams for spacing the rivets graphically would be much greater than in the analytical solution of the problem.

If any of the foregoing equations be transformed and solved for v, the resulting value for Maximum $s \times$ Minimum p determines the necessary thickness of web when reference is made to a table of rivet values for any particular size of rivet.

As to rivet spacing in stiffeners, these formulas may or may not apply. If they apply, it is necessary to assume that the stiffeners act as columns, and this, at least, is a moot question. To illustrate, imagine a number of planks piled on edge one upon another; unless supported laterally, they would not stand alone, much less carry a load imposed upon them; but, if posts directly opposite each other be placed on each side, in contact with the planks but not attached to them, a considerable load would be sustained by the planks, although receiving little assistance from the posts. This seems to show that the office of the stiffener is not to take stress, but to enable or compel the web to do so; and, for that reason, only a few rivets are needed in the stiffeners to properly hold them together. As usually constructed, there is no doubt that the stiffeners act with the web and serve to disturb and complicate the theoretical lines of stress, but there is no reason for assuming that because they take some stress they must take all over and above the buckling value of the web.

The foregoing reasoning applies specially to intermediate stiffeners, which do not serve to convey any external load to the web.

In the case of concentrated loads brought to the web from transverse floor beams, or more or less distributed on either flange of the girder, the stiffeners must be proportioned to convey these loads to the web, but in each of these cases practical considerations of construction will usually determine the size of stiffeners.

At the ends of the girder the lines of stress being cut, the web has little buckling value, and the stiffeners must be proportioned to convey practically the whole shear to the support. The stress in the end stiffeners due to shear varies practically from zero at the top to about the full amount of the shear at the bottom, and there must be rivets enough to convey this shear from the web to the stiffeners. A girder properly stiffened at the ends and at points of local concentrated loads will usually need very little intermediate stiffening.

The author suggests figuring stiffeners as flat-ended columns, and the questions naturally arise: Upon what will these columns rest? If on the bottom flange, how can rivets enough be grouped in the

flange at the base of the stiffeners to convey the reaction of the column from the flange to the web? And why should the shear follow the roundabout path through stiffeners and flanges? Mr. LaChicotte.

The author refers to a well-known formula for finding the buckling values of webs, and advocates the spacing of stiffeners about the depth of the girder apart whenever the shear in the web equals or exceeds the value of w in the formula

$$w = \frac{10\,000,}{1 + \frac{h^2}{3\,000\,t^2}}$$

This method of spacing stiffeners, while quite common, is not consistent, for the reason that this formula pretends to give the buckling value of the web, but does not express the relation of this value to the shear.

If the web needs stiffening at all, why not in some proportion to the shear? If the foregoing formula be applicable at all, it should be made to express the actual relation between load and column length by substituting "shear" for "w," and solving for h, whence results for the distance between stiffeners or flange angles,

$$h = \sqrt{\frac{30\,000\,000\,t^2}{\text{shear}} - 3\,000\,t^2};$$

When h is more than the depth of girder, no stiffeners are required, except for local concentrated loads.

The common practice of stiffening plate girders at distances apart of about the depth of the girders, is probably far on the side of safety; but it would, nevertheless, be unwise to depart materially therefrom, until a more rational method has been devised.

In regard to determining the size of stiffeners necessary to prevent the web from buckling, it may be said that no rational method of figuring the stresses in the webs of plate girders has yet been devised, and therefore no exact determination of the size of stiffeners is possible.

It is useless to say that if the web maintained its plane form, no stiffeners would be required; but the webs are not perfect planes, having many bulges or buckles of more or less magnitude as they come from the mills, and which buckles cannot always be detected or removed in the process of manufacture of the girder.

It may safely be said that stiffening angles large enough to accommodate the rivets and conform to the usual shop practice will prevent the web from buckling. Of course, the end stiffeners, as well as those at points of local concentrated loads, may be larger than elsewhere; and, perhaps, the most that can be said is that good judgment based on present practice will usually be a safe guide in special cases.

Mr. Nichols. O. F. Nichols, M. Am. Soc. C. E.—The speaker admits with candor, and with pride, that so far as he has had anything to do with plate girders, he has followed the teachings and experience of Theodore Cooper, M. Am. Soc. C. E., who taught that in order to get more rivets safely into the flange angles, and to devolop fully the efficiency of these angles, their depth should be increased so as to carry at least two rows of rivets wherever many rivets were required. This has a double advantage: The rivets need not be spaced dangerously near together, and the deeper angles confine all the elemental portions of the web, so that, considering them as columns, they are practically fixed at the ends, and, consequently, are better able to sustain the loads imposed.

The Pennsylvania practice has been very largely followed in this country and elsewhere. A good deal of the misunderstanding about stiffeners in plate girders is probably caused by following the Pennsylvania standard too closely. An interesting experiment was made by one of the Pennsylvania engineers: He constructed a model girder, from paper, then, after cutting the web on diagonal lines at 45° in one direction between the stiffeners, spaced at equal intervals about equal to the depth of the girder, the girder was put under stress. He found that the stiffeners took the vertical load, and that the elements of the web buckled under compression and remained straight under tension. Of course, cutting the web in that manner converted the model girder into a sort of Pratt truss, and the conclusion drawn, that stiffeners were necessary at intervals equal to the depth of the girder, was quite a logical sequence from the premises.

The more rational method, and that more closely followed in recent times, is to consider plate girders as acting very much as lattice girders act. Take a case in which the girder web is not stiffened; then, if it is regarded as a lattice girder, there is an infinite number of intersections, and if it be assumed that the stresses pass in diagonal lines through the web, in both directions, while the compression would tend to buckle the web, the tension along the other lines at right angles would tend to keep it straight. As a matter of fact, in all tests of plate girders, it is found that there is a remarkable and an unaccountable degree of stiffness in the unsupported web.

Mr. LaChicotte is right in stating that, if there are rivets enough in the stiffeners to prevent them from buckling out, it is all that can be required. Fitting them against the flanges is not necessary, unless, as he has said, the top of the girder is to sustain the load; in which case, of course, there must be a close fit, at least against the top angles.

There has been a great misuse of stiffeners. In some elevated railroad structures, for example, where the uniform loads are relatively low, and where the girder is often very shallow, a ⅜-in. web is generally

used with a multitude of stiffeners. In quite ordinary practice the angle is used as a stiffener, and is often placed with its back toward the middle of the girder. At the middle of the girder, either for symmetry or from a desire to be strictly impartial between the two ends of the girder, two stiffeners are placed, back to back, where none is needed. These structures have generally been built by the pound, on designs by the contractors, and in such cases this excess of stiffeners has an advantage not easily overlooked. Some ten years ago, several miles of elevated structure were built in which, by deepening the standard girder about 15% and omitting stiffeners, the average strength was increased and the weight diminished nearly 20 per cent. Of this structure, a leading bridge expert, recently engaged by new owners to report on its strength, said: "The bold omission of these (stiffeners) has apparently worked all right in practice, and secured a substantial economy of metal." Mr. Nichols.

Of late years there has been a most exemplary attempt to avoid serious errors in the use of stiffeners; to rely, indeed, almost entirely on the web itself, unaided by stiffeners; and now it is the exception, rather than the rule, to find stiffeners in stringers on large bridges, although they carry very heavy loads and are subject to varying stresses, of impact, etc. In the Buffalo Trace Viaduct, designed by Edwin Thacher, M. Am. Soc. C. E., some ten or twelve years ago, the longer girders are about 60 ft. in length and approximate 5 ft. in depth. The viaduct carries a standard-gauge railroad, and no stiffeners whatever are used, but the webs of the girders are $\frac{3}{8}$ in. thick.

Engineers are approaching a belief in simplicity of design, and that the fewer the actual number of members used, the better. If the metal in the web can be made to take the stresses directly, and without the aid of stiffeners, it is a great deal better, very largely because it is absolutely impossible to determine with the remotest degree of certainty how such stiffeners will act.

F. D. Rhodes, Jun. Am. Soc. C. E.—The speaker's experience in designing girders for buildings points conclusively to the advantage of using the graphical method for spacing rivets when the girders are subject to combinations of concentrated and distributed loads, or any kind of loading which gives an unsymmetrical moment diagram. Mr. Rhodes.

It has been contended that the author's method involves extra labor, as it necessitates the plotting of a shear diagram; but in cases of the kind just mentioned it is the speaker's belief that graphic statics will most quickly and accurately determine the length and position of the cover-plates, and that it requires but very little extra labor to plot the shear diagram directly off the load line of the force polygon.

Girders in buildings often carry walls or floor arches which are

Mr. Rhodes. supported directly by the flanges, for which cases the speaker's practice, in determining the shear values for the various pitches, differs from the author's, as is shown in Fig. 10, and the accompanying table. The example is taken from the design of a 36-in. girder, web $\frac{11}{16}$ in. thick, rivets $\frac{7}{8}$ in., with the top flange carrying a load of 9 tons per lineal foot.

RIVET DIAGRAM.

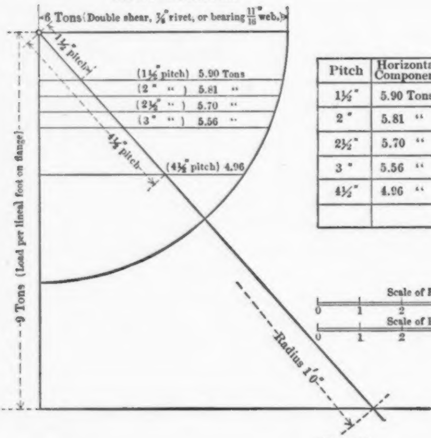


TABLE.

Pitch	Horizontal Component	Shear	Calculation
1½"	5.90 Tons	114 Tons	$\frac{5.90 \times 29}{1.5}$
2"	5.81 "	84.2 "	$\frac{5.81 \times 29}{2}$
2½"	5.70 "	66.1 "	$\frac{5.70 \times 29}{2.5}$
3"	5.56 "	53.7 "	$\frac{5.56 \times 29}{3}$
4½"	4.96 "	32.0 "	$\frac{4.96 \times 29}{4.5}$

$$S = \frac{r' \times h}{p}$$

$$h = 36'' - 7'' = 29''$$

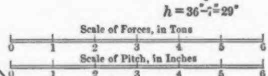


FIG. 10.

The horizontal components are scaled directly off the rivet diagram, the calculations are made with a slide rule, and the shears are then plotted on the shear diagram.

Mr. Jonson. ERNST F. JONSON, Assoc. M. Am. Soc. C. E. (by letter).—The author's method of spacing rivets in plate girders is undoubtedly the most convenient one yet proposed. There are, however, some points in the paper which require further consideration. In order that the problem of the plate girder may be better understood, it is necessary to find the actual stresses in the web.

These stresses may be found by the following graphical method:

Let s = total vertical shear;

m = unit shear at neutral axis;

t = increase of extreme fiber stress per unite of length of girder;

a = area of girder section;

w = thickness of web;

r = radius of gyration of girder section;

d = distance from neutral axis to extreme fiber;

b = distance from neutral axis to center of gravity of half girder section.

Mr. Jonson.

We know that $s = \dfrac{a\, r^2\, t}{d}$, or $t = \dfrac{s\, d}{a\, r^2}$.

We also know that $w\, m = \dfrac{a}{2} \dfrac{t\, b}{d}$.

Hence,
$$m = \frac{b\, s}{2\, r^2\, w} \quad \text{I}$$

and
$$s = \frac{2\, m\, r^2\, w}{b} \quad \text{II}$$

Draw a sectional view of the girder, Fig. 11, and construct a Mohr diagram of the square of the radius of gyration. This is done in the following way: Divide the cross-section of the girder into a number of parts (twelve in the figure). Take half the cross-section area as pole distance $C\,D$. Lay off the parts of one-half of the cross-section area on the line $C\,E$. Draw the moment diagram $B\,C\,G$ as if the parts of the cross-section area were forces acting in the center of gravity of each part. The horizontal ordinates of the curve $C\,G$ multiplied by the pole distance $C\,D$ give then the moment of the part of the cross-section area outside the point in which the ordinate is taken. Hence, $B\,G$ is the distance from the neutral axis to the center of gravity of the half cross-section; $B\,G = b$.

Since the horizontal bases of the triangles of which the area $B\,C\,G$ is composed represent the moments of the various elements of the cross-section divided by half that area, the triangles themselves must represent half the moment of inertia of these elements divided by half the cross-section area. Hence, the area $B\,C\,G$ is equal to half the square of the radius of gyration of the cross-section area with reference to the neutral axis; $B\,C\,G = \dfrac{r^2}{2}$.

Since the shear at any point in the web is proportional to the cross-section area outside of that point multiplied by the average bending stress acting in that area, *i. e.*, proportional to the moment of the area with reference to the neutral axis, the horizontal ordinates of the curve $A\,G$ are proportional to the shear.

The area $A\,B\,G$ is twice as large as the area $B\,C\,G$, because the trapezoids of which it is made up are twice as large as the corresponding triangles of which the area $B\,C\,G$ is composed; $A\,B\,G = r^2$.

If we now take the pole distance, $A\,F = b$, and draw the moment diagram $A_1\,C_1\,C_2$ with reference to C, we get $C_1\,C_2 \times A\,F = 2 \times A\,B\,G$, or $C_1\,C_2 = \dfrac{2\,r^2}{b}$. We also get the curve $A_1\,C_2$, the ordinates of which toward the axis $A_2\,C_2$ are proportional to the part of the double area $A\,B\,G$ which lies below the point in which the ordinate is taken. Hence these ordinates are proportional to the compression produced by a load on top of the girder. If we then lay off the load per unit of length divided by the web thickness $A_2\,N$, and draw, by means of

Mr. Jonson. parallel lines, the curve $N\,C_2$, with ordinates proportional to those of $A_1\,C_2$, $A_2\,N\,C_2$ is the diagram of the perpendicular compressive unit stress due to the load.

Now, lay off $C_1\,K = 10$, $C_1\,K_1 = 10$, and $C_1\,L = 10w$, and draw the parallel lines $K_1\,L$ and $K\,L_1$, thus making $C_1\,L_1 = \frac{10}{w}$. Lay off $C_1\,M = s$, and draw the parallel lines $C_2\,L_1$ and $M\,M_1$, making $C_1\,M_1 = 10\,m$. For $C_1\,M$ and $C_1\,M_1$, a smaller scale must be used. Lay off $B\,G_1 = m$, and draw, by means of parallel lines, the curve $A\,G_1$ with ordinates proportional to those of curve $A\,G$. These ordinates then represent the unit shear at the various points of the web.

Multiply $C_1\,C_2$ with $\frac{a\,b}{2}$ and lay off the result $A_2\,H = a\,r^2$. Also, lay off the bending moment $A_2\,T$, both to a smaller scale than that of the rest of the diagram. Draw the parallel lines $H\,T$ and $A_3\,C_3$. The triangles $A_2\,A_3\,B_2$ and $B_2\,C_2\,C_3$ are then the diagrams of the unit stresses due to bending; the ordinates of $A_3\,B_2$ representing the compression and those of $B_2\,C_3$ the tension, for the unit stress is to the distance from the neutral axis as the bending moment is to $a\,r^2$.

In order to find the resulting principal stresses* lay off the unit shearing stress on a vertical line from U to R. Take the point Q, dividing the distance $O\,U$ in two equal parts, as a center, and draw the circle $P_1\,R\,P_2$ through R. The two principal stresses are then the tension $P\,P_1$ acting in the direction $P_2\,R$, or $P\,S$, and the compression $P\,P_2$ acting in the direction $P_1\,R$, or $P\,S_1$. Hence, the ordinates of the curve $I_1\,J_1$ represent the tensile stresses, and those of the curve $I_2\,J_2$ the compression stresses.

From the diagram it will be seen that unless the unit shear is very small (in the present case less than one-third of the extreme fiber stress) the maximum stress is not at the extreme fiber, as generally supposed, but in the web at the upper and lower pitch lines.

The shear diagram further shows that the h, in the rivet formula quoted by the author, should be taken as the distance between pitch lines only when the formula is used for rivets in stiffeners supporting concentrated loads or reactions. When used for rivets in flange angles it is quite on the safe side to make h equal to the distance from back to back of angles. In the present case it is even greater than the total height of the girder.

The author holds (page 553) that when stiffeners are used, the web takes only a part of the shear, leaving the remainder to the stiffeners. This is clearly impossible, unless the stiffeners are arranged so as to form a complete lattice system. If, in the present case, a stiffener were inserted along the line $A\,C$ it would greatly reduce the vertical compressive stresses $A_2\,C_2\,N$, but would not in any way affect the shearing stresses. It would, however, increase the resistance of the

* Rankine, "Applied Mechanics," Ch. V., Sec. 3.

Mr. Jonson.

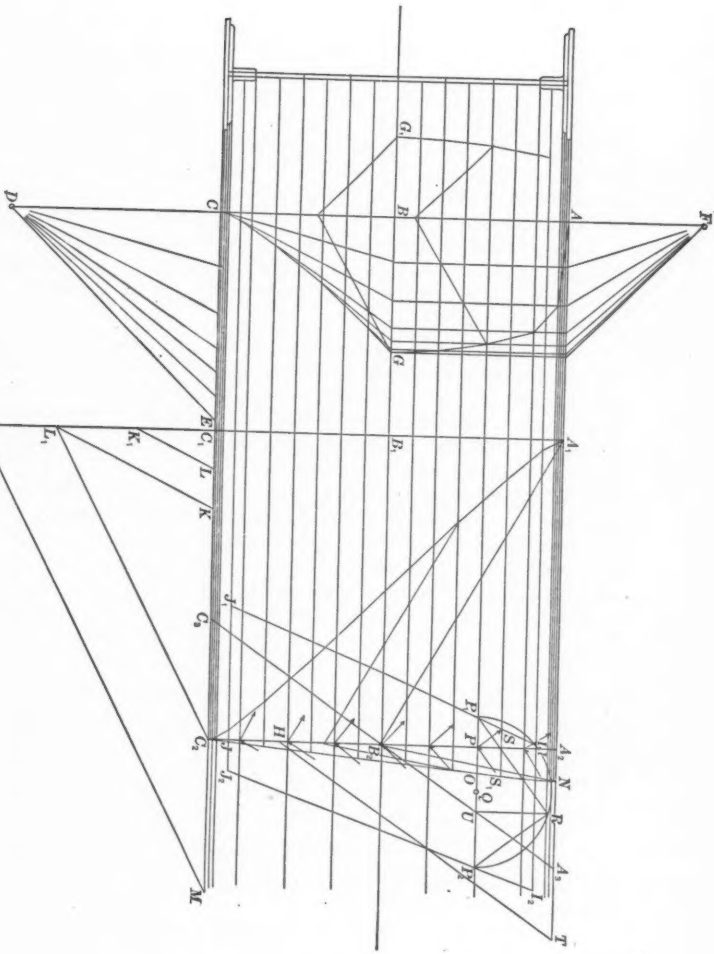


Fig. 11.

Mr. Jonson. central part of the web by preventing buckling or wrinkling. Unfortunately the theory of wrinkling has not yet been developed, so that the dimensions and spacing of stiffeners and their rivets cannot be calculated. The writer refers to stiffeners proper. Uprights, inserted to transmit a concentrated load or reaction to the web, should be regarded as columns loaded along their whole length and, therefore, subject to a stress proportional to the horizontal ordinates of the curve $A_1\ C_2$. Such uprights may, therefore, as the author says, be calculated for the allowed unit stress. The length, however, should not be more than 120 times the radius of gyration perpendicular to the web, or the length beyond which a column is good for less than one-half of the allowable unit stress of the material, because, in such uprights, the load at the center is one-half of the load at the end.

If the load were applied at the bottom instead of at the top the present diagram would have to be turned upside down. The curve $I_1\ J_1$ would then represent the compressive stress, and the curve $I_2\ J_2$ the tensile stress.

Mr. Wing. C. B. Wing, Assoc. M. Am. Soc. C. E. (by letter).—Methods of reducing the labor of computations, in designing bridge and other structures, are usually of most value to the person evolving the method. Therefore, it is with hesitation that the writer presents other methods for solving the problems, solutions of which have been presented so clearly in Mr. Schmitt's paper.

In the following graphical diagrams use is made of the principle that any equation involving three variable quantities may be readily solved for a large range of values of the variables by the use of co-ordinate paper with two sets of diagonal lines constructed to correspond to assumed values of two of the variables.

Thus, the diagram, Plate XI, Fig. 1, is used to solve the equation

$$v = t\,d\,C$$

in which d, t, and C are variables. v is the bearing strength of a rivet, of diameter d, bearing on a plate, of thickness t, and capable of resisting safely a bearing stress of C per unit of area.* The equation may be thrown into the form

$$C\,d = \frac{v}{t} \doteq X.$$

The co-ordinate paper is now used to find the product $C\,d = X$, and then, from this product, the value of v is found for the given value of t. Thus, on the left of the diagram, values of C from 9 000 to 26 000 lbs. per square inch, covering the range of ordinary practice, are laid off to scale as ordinates. Similarly, at the bottom of the diagram, values of

* v is the quantity termed "rivet value" on page 552. In what follows, the writer has adhered to the notation of the paper as closely as possible.

SAFE LOADS FOR RIVETS IN BEARING. $v = t\,d\,C$

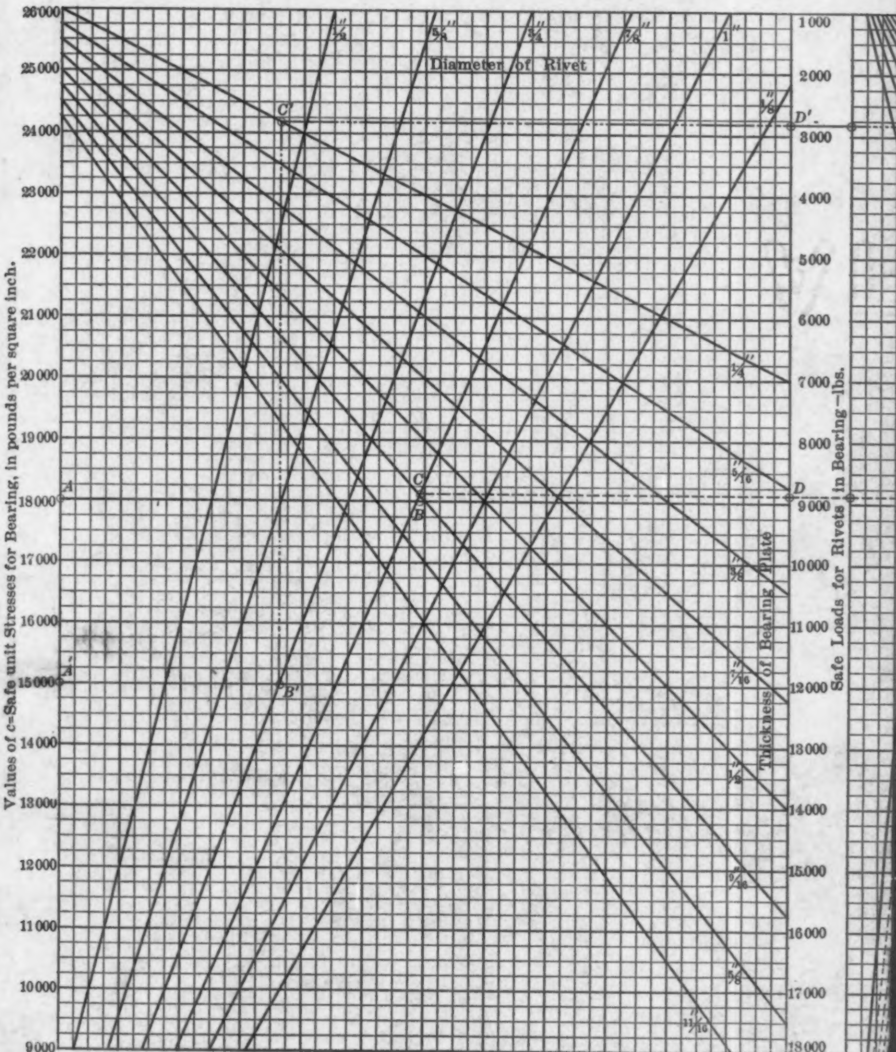


FIG. 1.

SPACING FOR RIVETS IN GIRDER FLANGES $p=\frac{vh}{S}$

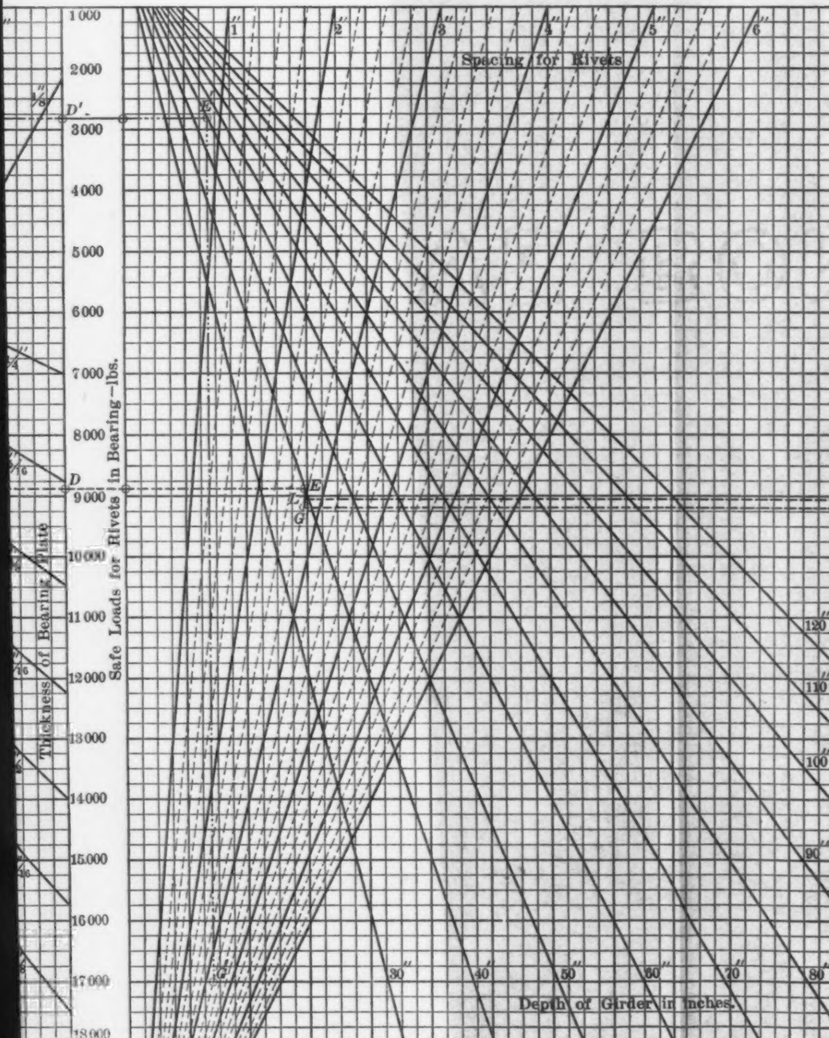


FIG. 2.

PLATE XI,
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WING ON RIVET SPACING.

SOLUTION OF EQUATION $S' = \sqrt{S^2 + w^2 h^2}$

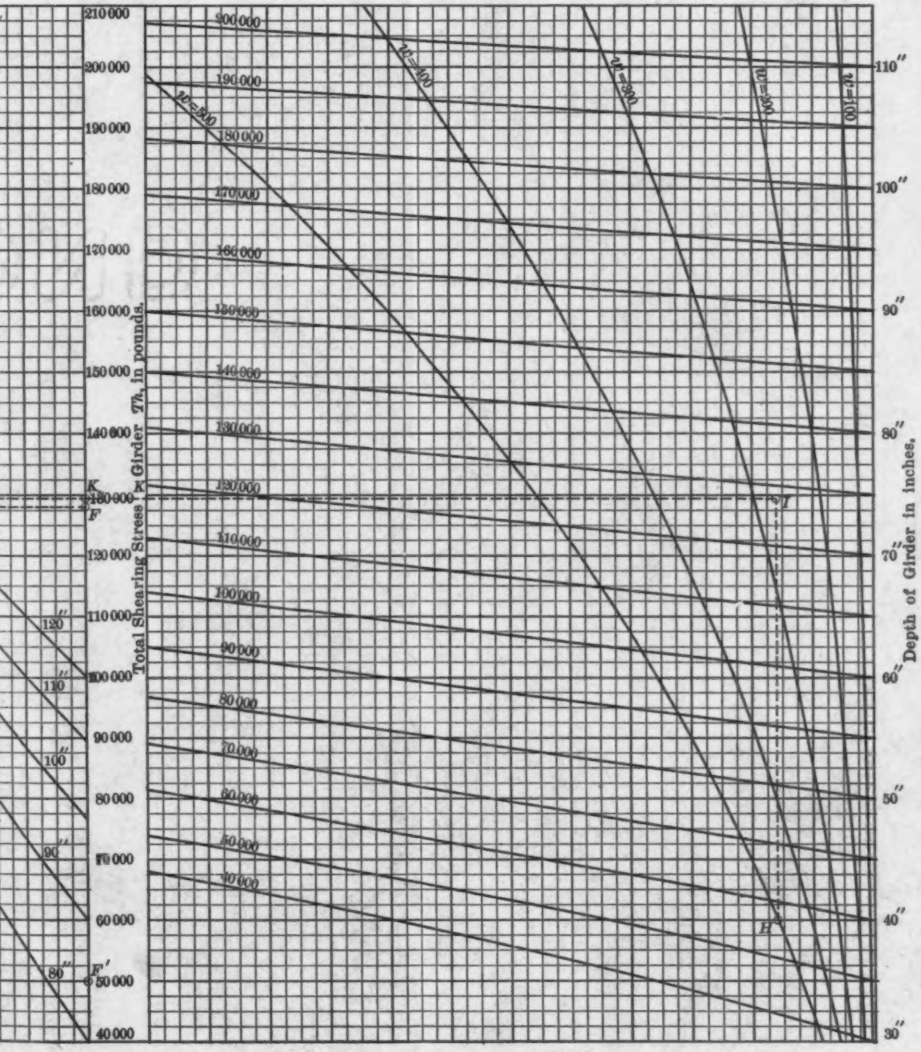


FIG. 3.

X, covering the range of values of the product $C\,d$, are laid off as abscissas to some suitable scale.* Then diagonal lines are drawn corresponding to values of d ranging from $\frac{1}{2}$ in. to $1\frac{1}{2}$ ins. With C and X as co-ordinates, the equation of any such diagonal, for a given value of d, is **Mr. Wing**

$$C\,d = X.$$

As the quantities in this equation are all of the first power, these diagonals are straight lines radiating from the intersection of the vertical and horizontal through the zero values of X and C, respectively; for, when $C = 0$, $X = 0$ for all values of d. Therefore, one other point on each diagonal is all that is required to determine its position on the diagram. To locate such points assume $C = 24\,000$. The following values of X result for the given values of d:

d	X
$\frac{1}{2}$	12 000
$\frac{5}{8}$	15 000
$\frac{3}{4}$	18 000
$\frac{7}{8}$	21 000
1	24 000
$1\frac{1}{8}$	27 000

On the horizontal through $C = 24\,000$ plot the foregoing values of X, and from these points draw straight lines to the point of intersection of a vertical through the zero of the scale of X with a horizontal through the zero of the scale of C. Points where these diagonal lines intersect horizontals through the scales of C will be vertically above the value $X = C\,d$ on the scale of X. Thus, to find the product $C\,d$ for any values of C and d within the range of the scales, follow the horizontal line through C to the right until it intersects the diagonal line corresponding to the value of d. The required product $C\,d = X$ will be found on the bottom scale vertically beneath this point.

In a similar manner, laying off on the right side of the diagram values of v, diagonal lines corresponding to values of t are drawn radiating from the intersection of a vertical and horizontal through the zero points of the scales of X and v, respectively. As before, one other point on each diagonal line is found from the equations of such lines, $X = \frac{v}{t}$, by assuming $X = 24\,000$ and plotting values of v corresponding to assumed values of t. Thus, to find the value of v corresponding to values of X and t, follow the vertical through the value of X until it intersects the diagonal corresponding to the value of t; the required value of v will be found on the scale of values of v horizontally to the right of this point.

Finally, with the diagram as constructed, to find the value of v corresponding to any values of t, d and C, within the limits of the scales, proceed as follows: Enter the diagram at the point on the scale at the left,

* The numerical values of X are not shown on the diagram, as they are not of practical value. The scale ranges from 4 000 on the left to 28 000 on the right.

Mr. Wing. corresponding to the value of C, move horizontally to the right to the point of intersection with the diagonal corresponding to the value of d,* from this point move vertically to the point of intersection with the diagonal corresponding to the value of t; the required value of v will be found on the scale of values of v horizontally to the right of this point.

Example.— $C = 18\,000$ lbs. per square inch, $d = \frac{7}{8}$ in., $t = \frac{9}{16}$ in. Required, the value of v. The diagram, Fig. 1, Plate XI, is entered at A, and, following the dotted lines through B and C, the value $v = 8\,860$ lbs. is found at D on the right-hand scale of values of v; or with $C = 15\,000$ lbs. per square inch, $d = \frac{3}{4}$ in. and $t = \frac{1}{4}$ in., entering the diagram at A', and following through B' and C' to D', the value $v = 2\,800$ lbs. is found.

The diagrams, Figs. 2 and 3, Plate XI, for the solution of the equations $p = \frac{v\,h}{S}$ and $S' = \sqrt{S^2 + w^2\,h^2}$ have been constructed in a similar manner.†

To facilitate this method of determining the spacing of rivets in girders, the equation given by the author on page 556, for the case in which the load carried by the girder is distributed uniformly to the flange angles, has been changed to the form

$$p = \frac{v\,h}{\sqrt{S^2 + w^2\,h^2}}$$

by the substitution of $w = \frac{Q}{k}$, in which w is the load on the flange per unit of length.‡

The diagram, Fig. 3, Plate XI, gives a solution of the quantity $S' = \sqrt{S^2 + w^2\,h^2}$. With this value, the value of p can be determined by Fig. 2, Plate XI.

The three diagrams on Plate XI, arranged in the order shown, give a ready means of solving graphically the problems ordinarily arising in practice concerning the pitch of rivets in girders. Thus, to determine the pitch of rivets in the flanges of a girder, at a point where the shear $S = 128\,000$ lbs., with $h = 40$ ins., $d = \frac{7}{8}$ in., $t = \frac{9}{16}$ in., and $C = 18\,000$ lbs. per square inch, Fig. 1, Plate XI, is entered at A on the left, as before, and the dotted lines followed to E, Fig. 2, which is the point of intersection of the horizontal through the rivet value v, with the diagonal corresponding

* Note that the vertical through this point gives the value of $Cd = X$, and that this value of X is to be used in finding the value of $v = Xt$.

† Note that in Fig. 3, Plate XI, the variable quantities in the expression $\sqrt{S^2 + w^2\,h^2}$ are of the second power; therefore the diagonal lines of the diagram are curves instead of straight lines, and several points on each curve must be plotted in order to construct it on the diagram. Except for this additional labor, the method of constructing the diagram is the same as for Fig. 1, Plate XI.

‡ If the inch and pound are used as units, w must be expressed in pounds per lineal inch.

to a value of $h = 40$ ins. If the load carried by the girder is not distributed to the flange angles, the required pitch of rivets, p, is found by following the vertical through E down to the point G horizontally to the left of the point F, which point corresponds to a value of $S = 128\,000$ on the scale on the right of Fig. 2, Plate XI. This point G is found to lie very near the diagonal line for values of $p = 2\frac{3}{4}$ ins.; therefore, $2\frac{3}{4}$ ins. is the pitch required. Mr. Wing.

If the load carried by the girder is distributed to the flanges, with $w = 500$ lbs. per lineal inch, Fig. 3, Plate XI, is entered at H, the point of intersection of the diagonal corresponding to $w = 500$, and the horizontal through $h = 40$ on the scale on the right of the diagram; the vertical through this point is followed to I, a point corresponding to $S = 128\,000$, as shown by the transverse diagonals. $S' = 129\,500$ is found horizontally to the left of this point at K. The required pitch, p, is determined by the location of the point L, and is found to be 2.7 ins. Practically, spacings varying by less than $\frac{1}{4}$-in. are seldom used; therefore, $p = 2\frac{1}{2}$ ins. is the required spacing of the rivets in the flanges of a girder under these conditions.

Similarly, for $C = 15\,000$, $d = \frac{3}{4}$, $t = \frac{1}{4}$, $h = 60$, $S = 50\,000$, and $w = 100$, the required pitch is found to be $3\frac{1}{4}$ ins., by entering the diagram at A', and following the dotted lines through B', C', D', E', to G'. Referring to Fig. 3. Plate XI, at the point of intersection of the horizontal through $h = 60$ and the diagonal $w = 100$, it is seen at once that the fact that the load is uniformly distributed to the flange angles will not appreciably change the spacing of the rivets.

Fig. 3, Plate XI, shows clearly that, in many of the cases arising in practice, neglecting to consider the distributed load would cause but a slight error. It may be well to note, in this connection, that the formula

$$p = \frac{v\,h}{\sqrt{S^2 + w^2\,h^2}}$$

is only approximately correct. Thus, considering a section of a girder between two rivets as a free body in space in equilibrium under the forces acting upon it: On the assumption that the moment is carried by the flanges at the line of the pitch of rivets, the reactions of the portions of the girder removed will be as shown by the arrows in Fig. 12. The body being in equilibrium, the sum of the moments about any point, as the lower right-hand rivet, must equal zero, *i. e.*,

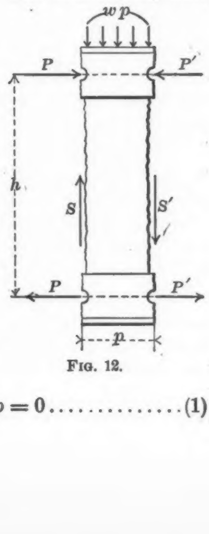


FIG. 12.

$$[P' - P]\,h + w\,p \times \frac{p}{2} - S\,p = 0 \ldots\ldots\ldots\ldots (1)$$

Mr. Wing. Next, considering the forces acting on one of the top rivets, Fig. 13, the following results:

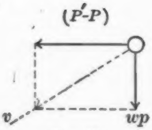


FIG. 13.

$$[P' - P]^2 + w^2 p^2 = v^2$$

$$\text{or } [P' - P] = \sqrt{v^2 - w^2 p^2}.$$

Substituting this value in Equation (1), the value of the required pitch of rivets is found to be

$$p = \frac{h\,v}{\sqrt{[S - \frac{1}{2}\,w\,p]^2 + w^2\,h^2}}$$

$$\text{instead of } p = \frac{h\,v}{\sqrt{S^2 + w^2\,h^2}} \qquad (2)$$

The approximate value given by Equation (2) is on the side of safety, and practically is sufficiently exact. In fact, as before stated, the error will be slight, in most cases, if the distributed load is entirely neglected.

Figs. 14, 15 and 16 have been constructed by the same methods used in constructing Plate XI. Fig. 14 replaces the author's Tables Nos. 1 and 2 and gives a solution of the equation

$$P = \frac{C\,h\,t}{1 + \frac{1\,h^2}{3\,000\,t^2}}$$

in which C has values varying from 10 000 to 18 000;

h = depth of girder;

t = thickness of web plate;

P = safe shearing load for a web plate without stiffeners.*

Nine tables similar to Tables Nos. 1 and 2 would be required to cover the range of values of C given in Fig. 14. Results can easily be read to the nearest 500 lbs., which practically is as close as required. Thus, if $h = 40$, $t = \frac{9}{16}$, and $C = 10\,000$, the diagram is entered on the left at A, with the value $h = 40$ as an ordinate, the horizontal line through this point is followed to the right until it intersects the diagonal line corresponding to a value of $t = \frac{9}{16}$, at B; from this point the vertical line is followed to its intersection at C with the diagonal corresponding to a value of $C = 10\,000$. The value of the safe total shear, 84 000 lbs., is found horizontally to the right of this point at D on the scale of values of P. Similarly, for $h = 60$, $t = \frac{3}{8}$, and $C = 10\,000$, the value of $P = 24\,000$ lbs. is found.

If the total shear on the girder is greater than the safe total shear given above, stiffeners will have to be used, and the safe total shear, or the required thickness of web plate, determined by Fig. 15.

The method of deriving the formula solved by Fig. 14, will be found in Church's "Mechanics of Materials," page 383. In spite of testimony against the necessity of using stiffeners, the writer still believes that the best results will be obtained by their use when required by the

* The author's notation has been changed in this instance, in order that it may more closely correspond to the notation used in the preceding discussion.

Mr. Wing.

SAFE TOTAL SHEAR FOR WEB PLATES WITHOUT STIFFENERS.

$$P = \frac{Cht}{1 + \frac{1}{3000}\frac{h^2}{t^2}}$$

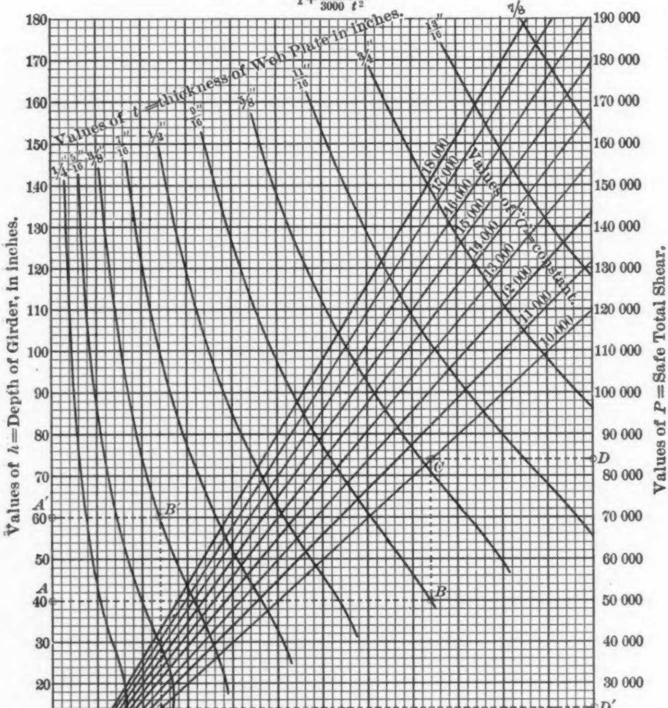


Fig. 14.

Mr. Wing.

SAFE TOTAL SHEAR FOR WEB PLATES WITH STIFFENERS.

$P = Sht$

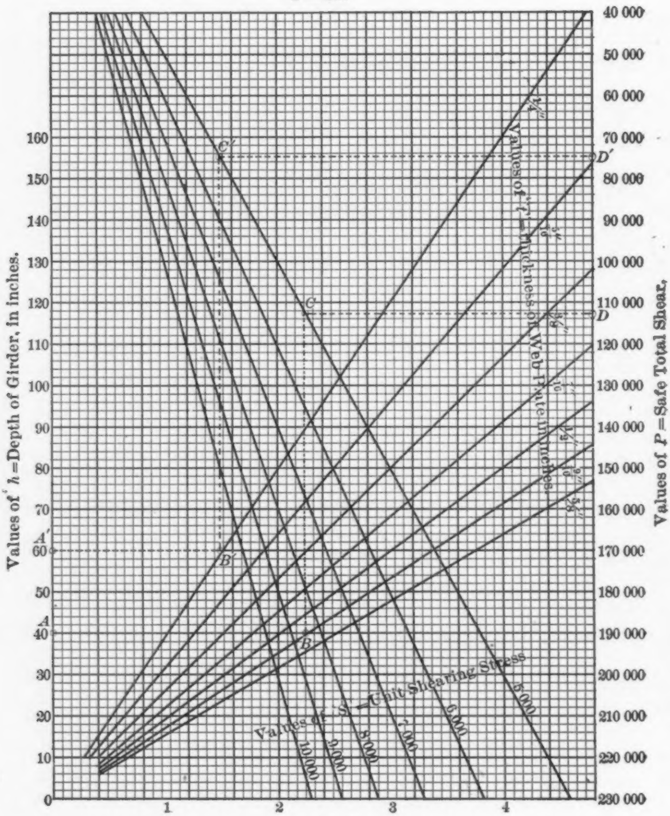


FIG. 15.

Mr. Wing.

SAFE LOADS FOR RIVETS, IN SHEAR, $P=\frac{\pi d^2}{4}S$

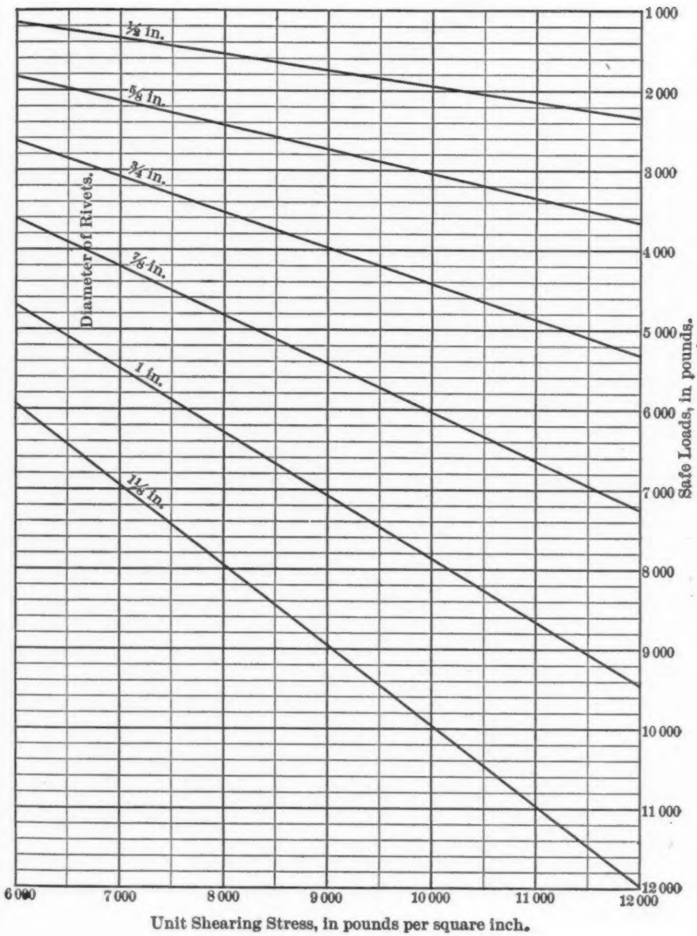


Fig. 16.

Mr. Wing. foregoing formula. When so used, they should be spaced at distances apart equal to the depth of the girder, and riveted securely to the web plate. The size of stiffeners and the number of rivets used in fastening them to the web plate should depend upon the judgment of the designer, and not upon calculation. If calculation is resorted to, it would be on the side of safety to design the two stiffeners with a reasonable amount of web plate as carrying the same stress as the post of a Pratt truss, similarly placed; that is, the total shear at that section should be carried by the stiffeners and a portion of the web plate acting as a column.

Fig. 15, giving safe shearing loads for web plates with stiffeners, and Fig. 16, giving the shearing values of rivets, have been constructed in a similar manner to the diagrams already described.

The foregoing methods for the graphical solution of equations are of wide application, and, in practice, are of great value as labor-saving devices. In general, a single graphical diagram will give a greater range of values than several pages of tables. The results, however, with ordinary scales, are only approximately correct to three places. The required accuracy of results, therefore, will usually determine whether the more convenient graphical diagram can be used as a substitute for a set of tables.

On page 562 the author gives an empirical rule for determining the depth of girders. If, as the author assumes, no restrictions are placed on the depth of a girder for a given purpose, it is desirable to use a depth which will carry the given load with a minimum weight of material in the girder. This "economic depth" is very closely given by the rule that the area of the flanges equals the area of the web plates, or

$$h = \sqrt{\frac{2\,M}{Tt}}\text{*}$$

in which h = depth of girder;

M = maximum moment carried by the flanges;

T = safe unit stress for flanges;

t = thickness of web plates.

This formula, while theoretically correct in form, is based on the assumption that there is no variation in the cross-section of the girder, and does not make allowance for the weight of details which do not change with the depth of girder.

In practice, a depth about 10% less than that given by the formula will be found to give the least weight for a girder capable of carrying a given load.

It seems to the writer that the rule given by the author must result in girders which are uneconomical and which have excessive deflections.

* See Johnson and Turneaure's "Modern Framed Structures," and an article by Henry Szlapka in *The Engineering Record*, March 27th, 1897, for more elaborate discussions of this formula.

E. Schmitt, Assoc. M. Am. Soc. C. E. (by letter).—Mr. Schaub's statement, deduced from experiment, concerning the erratic behavior of the webs in through and deck-plate girders is of much interest. Mr. Schmitt.

In what follows, the writer endeavors to approach the propositions (at least one of them) numerically. An example, arbitrarily chosen, will serve for the investigation, in which the following assumptions have been made:

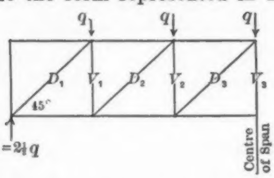
1. The sum of the web stresses in a panel acts along the center lines of imaginary web members of a square-paneled truss, with single diagonals in the panels, the length of each panel being equal to the depth of the truss.*

2. The loads (shear stresses) transmit themselves by the shortest route from the center of the span to the abutments.

3. The same factor of safety is to be maintained at all points in the web. Web members under compressive shear are to be safe against buckling. Compressive and tensile shears are to be considered as compression and tension stresses.

4. The web is to have the same thickness in the two cases.

Beginning with the case of the deck bridge, the imaginary truss will take the form represented in Fig. 17; the position of the diagonals



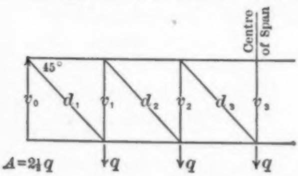
$$D_1 = -A\sqrt{2} \qquad V_1 = A - q$$
$$D_2 = -(A-q)\sqrt{2} \qquad V_2 = A - 2q$$
$$D_3 = -(A-2q)\sqrt{2} \qquad V_3 = A - 2\tfrac{1}{2}q = 0$$

Fig. 17.

enabling the loads to be transmitted "by the shortest route." The stresses obtaining in the web members are noted at the right of the diagram.

The diagonals, only, being in compression, the web will have to be proportioned for the stress obtaining in them.

In the case of the through bridge (or, perhaps, better expressed "trough bridge"), the imaginary truss will be represented by Fig. 18,



$$d_1 = +A\sqrt{2} \qquad v_1 = -(A-q)$$
$$d_2 = +(A-q)\sqrt{2} \qquad v_2 = -(A-2q)$$
$$d_3 = +(A-2q)\sqrt{2} \qquad v_3 = -(A-2\tfrac{1}{2}q)$$

Fig. 18.

* With the principal stresses acting at 45°, according to the theory of the plate girder, and with the loads concentrated at panel lengths, as assumed, and the further assumption that compressive web stresses will follow the line of least resistance, the arrangement of the direction of the forces, as shown in Fig. 17, suggests itself first. Fig. 18 follows by reversing Fig. 17.

Mr. Schmitt. permitting, in a similar manner to that in the first case, the loads to travel by the most direct route from the center of the span toward the abutments. The stresses obtaining in the web members are noted at the right of the diagram. They are of the same magnitude as in the corresponding members of the first case, but opposite in character. The diagonals being in tension and the verticals in compression, the stresses obtaining in the latter will, therefore, govern the dimensioning of the web for buckling.

Before comparing the compressive stresses per unit of length along the respective web members of the two classes of girders, first determine the ratio of the permissible buckling stress in the diagonals (in compression), of the deck bridge, with the one taking place in the verticals (also in compression), of the through bridge. For this purpose select the empirical formula given by Rankine.

Calling the reduced stress for buckling,

for the diagonals of the deck bridge, P^2.. (length of diagonals $= h\sqrt{2}$)

" verticals " through " , p,.. (" " verticals $= h$),

we have, $$P = \frac{k}{1 + c\left(\frac{h\sqrt{2}}{w}\right)^2} \text{ and } p = \frac{k}{1 + c\left(\frac{h}{w}\right)^2},$$

in which,
k = simple compression unit;
h = depth of girder;
w = thickness of web;
$c = \frac{1}{5\,000}$.

According to the specification, cited by Mr. Schaub, to avoid the use of intermediate stiffeners, assume the web to have a thickness of $w = \frac{1}{50}$ of the depth of the girder.

Introducing the given quantities in the equations for P and p, we obtain, by extending and simplifying, the denominators only of the two fractions,

$$P = \frac{2\,500\ k\ w^2}{2\ h^2};\quad p = \frac{5\,000\ k\ w^2}{3\ h^2},$$

and dividing the two values, the ratio is found to be:

$$\frac{P}{p} = \frac{2\,500 \times 3\ h^2}{5\,000 \times 2\ h^2} = \frac{3}{4}$$

or, the permissible compressive unit web stress in the deck bridge should be but three-fourths of the stress in the through bridge, due to the difference in length of the members.

Since the web is of the same thickness in the two girders, the foregoing statement would mean that the web in the deck bridge is stressed four-thirds as much as the web of the through bridge, for the same load, and only as far as buckling is concerned.

Comparing now the ratio of the actual stresses, per unit of length, in the respective representative web members of the two bridges, we will first ascertain, which diagonal (stress) of the deck bridge should be compared with the corresponding vertical (stress) in the through bridge for any of the panels. Mr. Schmitt.

Referring to Fig. 17, it is evident that for the panel at the center the compressive stress D_3 will be the governing one, for the whole panel; its influence will extend from the center of the span to the vertical V_3.

In the case of the through bridge, Fig. 18, however, the computation of the web for buckling will begin at the vertical v_2; v_2 being in compression. Reasoning similarly as regards the operation of stresses in the other, related, panels, we can say:

TABLE No. 4.

In the deck bridge, in.............	1st....	2d......	3d panel.	
compute web for comp. stress....	D_1...	D_2..........	D_3	(length of members, $h\sqrt{\ }$)
In the through bridge, at vertical	1	2	3	
compute web for comp. stress...	v_1....	v_2.......	$v_3 = 0$ (or d_3, tension)	(length of members, h.)

The sphere of action of the vertical, compressive stresses in the through bridge is represented in Fig. 19. Any ordinate in any of the triangles represents the columnar lengths of the web, subject, in turn, to the stresses v_0, v_1, v_2.

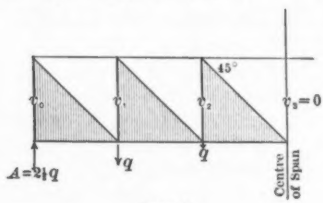


FIG. 19.

It will be seen that at v_0, v_3 and v_0, the critical points exist for web computation, and that it is permissible to compare the panel stresses, as given in Table No. 4.

Commencing with the stresses of the second panels of Table No. 4, the magnitudes of which are noted in Figs. 17 and 18, we have,

Simple comp. stress, per unit of length, along web, or diagonal D_2 $\Big\}$ $D_{u2} = -\dfrac{(A - q)\sqrt{2}}{h\sqrt{2}}$

Simple comp. stress, per unit of length along web, or vertical v_2 $\Big\}$ $v_{u\,2} = -\dfrac{(A - 2\,q)}{h}$

and $\dfrac{D_{u\,2}}{v_{u\,2}} = \dfrac{A - q}{A - 2\,q}$; $A = 2\frac{1}{2}\,q$,

we find, $\dfrac{D_{u\,2}}{v_{u\,2}} = 3.$

Previously, it was found that, for equal thickness of the webs in the two bridges, the stress in the web of the deck bridge was four-

Mr. Schmitt. thirds as much as in the web of the through bridge, and that, therefore, the actual ratio of the stresses per unit of length of D_3 and v_2 will be

$$\frac{D_{u\,2}}{v_{u\,2}} = 3 \times \frac{4}{3} = 4.$$

Similarly, it is found, in the first panel, at the abutment,

$$D_{u\,1} = -\frac{A\sqrt{2}}{h\sqrt{2}}.$$

$$v_{u\,1} = -\frac{(A-q)}{h},$$

and the simple stress ratio $\frac{D_{u\,1}}{v_{u\,1}} = \frac{A}{A-q} = \frac{5}{3}$,

and the actual stress ratio $\frac{D_{u\,1}}{v_{u\,1}} = \frac{5}{3} \times \frac{4}{3} = 2\,\frac{2}{9}$.

In the third panel, or that at the center, compare D_3 (compression), with d_3 (tension). Both stresses are of the same intensity, and both members are of the same length.

Referring to the formula for permissible unit stresses in the diagonals, we will have for D_3, introducing for w, $\frac{1}{50}\,h$,

$$P = \frac{2\,500\ k\,w^2}{2\,h^2}.$$

$$P = \frac{k}{2}.$$

Assuming the permissible unit stress for tension, also, as k (no reduction being necessary to be made for this kind of stress), it is evident that, for equal thickness of web, in the two girders (loads being the same), the web in the panel at the center of the deck bridge will be stressed twice as much as the web at the end of the same panel of the through bridge.

Mr. Schaub's deductions from his experiments, concerning the contrary behavior of the webs in the two classes of plate-girder bridges, seem, therefore, to be verified in a measure, and it could not, perhaps, be otherwise. It occurs to the writer, however, that what Mr. Schaub desires to prove by experiment can be explained by saying that, until now, the "theory of bridges" has failed to recognize the two classes of bridges (deck and through) in plate girders, although having distinguished these in trusses long ago.

The exceptions, made by the several discussors, principally those regarding the problem of the stiffeners, were, to a degree, anticipated.

The (theoretical) solution of this problem, it seems, could be effected in two ways: One would be, to figure the web for shear, then space the stiffeners according to the formula presented by Mr. La Chicotte,* to

* This formula is not new; see notes by Professor Swain (Mass. Inst. of Technology), which the writer had opportunity to inspect, before writing the paper.

prevent the buckling of the web. The other method would be, to do away with intermediate stiffeners and to figure the web for buckling. The writer, however, will repeat, with Mr. La Chicotte, that, perhaps, the most that can be said about this matter, is that good judgment, based on present practice, will usually be a safe guide in special cases. Enough has been said regarding "present practice" in the discussion; a repetition seems superfluous. Mr. Schmitt.

Mr. Nichols' remark, that it would be better to use flange angles so as to carry two rows of rivets, instead of one row, in which case the depths of the girder would have to be increased (under certain previously mentioned circumstances), is to be noted by designers as another serviceable constructive expedient, of advantage in more than one direction, and the writer has made use of it.

It conflicts, however, with the desirable proposition to obtain a maximum section modulus with the least amount of material. The material composing the flange area should be placed as far from the neutral axis as practical considerations will allow.

Mr. Jonson's demonstration of the internal stresses is very interesting and instructive. The opinion, however, that stiffeners should be arranged so as to form a complete lattice system, differs from that of the writer, who has always held that stiffeners need be used in a vertical position only, but, if required, should be spaced uniformly along the web. This arrangement converts the plate girder into a truss, the web performing the duty of the diagonals, subject to either compressive or tensile shear, as the case may be.

The writer's object was, not to present a complete or a new theory of the plate girder, but to exhibit a comprehensive, uniform and practical method of spacing the rivets, and of determining the point, from which on, toward the abutments, stiffeners are needed; including the reciprocal operations connected with the problem.

The method shown seems to him to be simply a direct property of the shear curve, in a developed form, and should find a good application in the presentation of the theory of the plate girder in connection with its construction.

The use of special methods and diagrams for working out the same problem is a personal matter. It is a privilege and a source of information to become acquainted with other roads than one may have been accustomed to travel. In this respect, the writer acknowledges that he has profited by the discussion.*

Mr. Wing's graphical tables for solving the problems connected with the design of the web of plate girders cover, very likely, all the cases that may arise in present-day practice.

The writer believes that when the numerical tables, Nos. 1 and 2, giving the total buckling values of webs, for 10 000 and 13 000 lbs.

* The foregoing remarks were written before Mr. Wing's discussion was received.

Mr. Schmitt. unit shearing stresses per square inch, are properly used, no urgent need exists to add seven more similar tables, to cover the range of values given in Fig. 14.

Let it be required, for instance, to find the total buckling values for the unit shears of 11 000, 14 000 and 18 000 lbs.; the procedure would be as follows :

For 11 000 lbs.	unit shear,	add to values of	Table	No. 1, one-tenth of the same;			
For 14 000 "	"	"	"	No. 2,	"	values of No. 1;	
For 18 000 "	"	"	"	No. 2, one-half	"	" "	etc.

Proceed similarly with the rivet tables.

The writer will not enter upon the question of the rapidity and ease with which the path of the several lines leading to final results can be followed through the different diagrams Mr. Wing has presented, and in the course of which a number of turning points occur. He would claim, however, that with his method the two main points, noted in the heading of the paper, are determined *in loco;* that is, the rivet pitch and the stiffener point are produced upon the girder itself, when once the shear curves for the loading have been plotted. This the writer considers an achievement to be appreciated for its clearness and directness.

The inexactitude of the rivet formula for compound shearing stresses is properly dismissed by Mr. Wing as unimportant; yet it must be stated that this formula, as well as the simple rivet formula, are both incorrect. This is because no account has been taken of the fact that the web takes up part of the bending moment, so that the stress to be carried by the rivets connecting the flange to the web is less than the two formulas assume. The number of rivets required, therefore, is also less, and the pitch is increased.*

To complete the paper, both sets of formulas for rivet pitch are given here:

For simple shear stress, the usual formula for pitch is,

$$p = \frac{v\,h}{S}$$

For simple shear stress, the correct formula for pitch is,

$$p = \frac{v\,h}{S}\,\frac{F + \frac{A}{8}}{F}$$

For compound shear stress, the usual formula for pitch is,

$$p = \frac{v\,h}{\sqrt{S^2 + \left(Q\,\frac{h}{k}\right)^2}}$$

* See Johnson and Turneaure's "Modern Framed Structures," pages 298, 305 and 306.

For compound shear stress, the correct formula for pitch is, Mr. Schmitt.

$$p = \frac{v\,h}{\sqrt{\left(S\,\frac{F}{F+\frac{A}{8}}\right)^2 + \left(Q\,\frac{h}{k}\right)^2}}$$

In which $F =$ area of one flange section;
$A =$ " the web "

The correct formulas can be somewhat simplified by expressing the area of the web in terms of the area of the flanges. According to the "Working rule for economic depth,"* the sum of the areas of the flanges ought to be from $1\frac{1}{2}$ to 2 times the area of the web plate.

Now, when $2\,F = \frac{3}{4}\,A$, we have $\frac{A}{8} = \frac{1}{3}\,F$

and " $2\,F = A$, " $\frac{A}{8} = \frac{1}{4}\,F$, and for:

Simple shear, correct formula for pitch,

$$p = \frac{4}{3} \text{ to } \frac{5}{4}\,\frac{v\,h}{S}$$

Compound shear, correct formula for pitch,

$$p = \frac{v\,h}{\sqrt{\left(\frac{3}{4} \text{ to } \frac{4}{5}\,S\right)^2 + \left(Q\,\frac{h}{k}\right)^2}}$$

In Figs. 1, 2, 3, 4 and 5, therefore, it is only necessary to lay off, instead of h, $\frac{F+\frac{A}{8}}{F}\,h$, for the case for simple shear; and to decrease, graphically, the ordinates of the main shear curves, in the ratio $\frac{F}{F+\frac{A}{8}} \times S$, for the case of compound shearing stresses, to obtain more exact results than those with which the ordinary theory is satisfied. The writer abstained from mentioning the correct formulas for rivet pitch, at the time, because he did not wish to overburden the presentation of his method, which is not affected thereby, and, further, for the reason that their acceptance in actual practice is problematical. The approximations all tend toward greater safety.

Mr. Wing's apprehension that the empirical rule† given by the writer would lead to uneconomical girders, having an excessive deflection, is too general an assertion, and should not be accepted unreservedly.

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* See Johnson and Turneaure, page 300, Section 282.

† Mr. Wing evidently refers to the first one, which is, $h = 6\,\frac{L}{10} + 3$ ins (h in inches, L in feet), page 562.

Mr. Schmitt. This rule is to be used, as was stated in the paper, in ordinary building practice only; that is to say, in lighter structural work, and not in heavier, and railroad work.

The expression, "if no restrictions are placed on the depth of the girders," must not be interpreted too literally; for, however contradictory this may sound, restrictions to depths of girders in buildings will always exist. Interferences with door and window openings have to be guarded against constantly, and the projection of girders beyond the ceiling line into rooms below has to be looked out for, also, with the same care.

Mr. Wing is, perhaps, unaware of the results, on the other hand, to which the formula he cites for economic depths will lead, if used in "ordinary" building practice.

The formula for economic depth cited by Mr. Wing

$$h = \sqrt{\frac{2\,M}{T\,t}}\text{*}$$

presumes for its application a "practical" thickness of web plate, say ¼ in. for building work and ⅜ in. for bridge work.

In heavier work, when indeed no restrictions whatever hinder the designer, this formula will give proper results, and the economic depth found thereby, for a particular case, should be aimed to be realized, if possible.

Concerning Mr. Szlapka's formulas† for economic depths, it should be noted that they are based on different assumptions, and derived in another manner.

The depths he finds are about the same as those mentioned by the writer. (Page 562.)

Mr. Szlapka's formulas cover four cases:

Case I.—One-ninth of web allowed as flange area; no cover plates, $h = \frac{1}{8} L$;

" II.—No allowance made for web; no cover plates, $h = \frac{1}{7} L$;

" III.—One-ninth of web allowed as flange area; girder with cover plates, $h = \frac{1}{9} L$;

" IV.—No allowance made for web; girder with cover plates, $h = \frac{1}{8} L$.

As to the excessive deflections presumed by Mr. Wing to take place in girders, the depths of which are derived by the formula,

$$h = 0.6\,L + 3 \text{ ins.},$$

the writer will state that they are well within the safe limit established by the standard rule governing them in building practice.

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* See page 588. It may be mentioned that Winkler finds $h = \sqrt{\frac{3\,M}{T\,t}}$. See his "Theory of Elasticity and Strength," Prague, 1867, page 220.

† *The Engineering Record*, March 27th, 1897.

This rule states that the deflection of a beam or girder shall not exceed $\frac{1}{360}$ of the length of the span, or $\frac{1}{30}$ in. per foot of span.* Mr. Schmitt.

This rule, in algebraic form, reads,

$$h = 0.497\ L; \text{ or, better, } h = 0.5\ L,$$

in which h is in inches and L in feet. Although this rule is looked upon as a thoroughly reliable and safe one, being, in fact, a standard in building practice, and recommended as such to architects as a safe guide; it will be found, nevertheless, that in their beam tables all the mills differ from it in excess.† This leads to the adoption of deeper and heavier beams than circumstances require. It is evident that the excess of material put into floor beams in this manner will be a considerable item, particularly in the modern type of tall buildings.

The derivation of this formula is very simple. From applied mechanics it is known that a girder supported at the ends, uniformly loaded, and with constant cross-section, has a deflection (in inches) $= \frac{5}{384}\ \frac{ql^2}{E\ I}$;

wherein, $E =$ modulus of elasticity $=$ 29 000 000 lbs. per square inch ;
$I =$ moment of inertia of the cross-section ;
$q =$ load per inch of length ;
$l =$ span of beam or girder, in inches.

It is known, further, that the bending moment and moment of inertia have the following relations:

$$M = \frac{2\ I\ T}{h}, \text{ or } I = \frac{M\ h}{2\ T}$$

also,

$$M = \frac{q\ l^2}{8}, \text{ whence } I = \frac{q\ l^2\ h}{16\ T}.$$

Introducing this last term in the formula for deflection,

$$\text{deflection} = \frac{5\ T\ l^2}{24\ E\ h}.$$

As stated before, this deflection should be $\leqq \frac{1}{360}\ l$; equating now, we obtain

$$\frac{l}{360} = \frac{5\ T\ l^2}{24\ E\ h},$$

introducing in this equation the known quantities, $T =$ 16 000 lbs., $E =$ 29 000 000 lbs.,

we obtain, $h = \frac{12}{290}\ l$........................(h and l both in inches)

* This rule is mentioned in all mill handbooks and treatises on building construction, and the writer thinks Tredgold was its originator. No rule, limiting the deflection of railroad bridges, seems to exist.

† Except the tables in the "Handbook of the Cambria Steel Company."

Mr. Schmitt. If it is desired to derive the depth of the girder in inches, when the span L is given in feet, then, put $l = 12\ L$;

whence, $$h = \frac{144}{290}\ L = 0.497\ L\ ;$$

or, better, $$h = 0.5\ L.$$

In words, one-half of the number which expresses the span of the girder in feet gives the depth in inches for a beam which will not deflect more than $\frac{1}{360}$ of its span.

Dividing, now, the exact depth required by the depth determined by the empirical rule, their ratio taken inversely will be the ratio of the deflections for the two cases.

We have (all dimensions in inches),

exact depth, $$h = 0.5\,\frac{l}{12},$$

empirical " $$h = 0.6\,\frac{l}{12} + 3,$$

and $$\frac{\text{Empirical deflection}}{\text{Exact deflection}} = \frac{0.5\,\frac{l}{12}}{0.6\,\frac{l}{12} + 3},$$

or reduced, $$\frac{\text{Empirical deflection}}{\text{Exact deflection}} = \frac{0.5\,l}{0.6\,l + 36}$$

Assuming, now, for example, $L = 30$ ft., $= 360$ ins., the foregoing ratio would be $= \frac{5}{7}$, and the limiting deflection, in terms of the length of the span, would be,

by the exact formula, $$= \frac{1}{360}\,l,$$

" empirical " $$= \frac{5}{7} \times \frac{1}{360}\,l = \frac{1}{504}\,l.$$

Even if the constant 36 is neglected, in the general formula for ratio of deflections, the last fraction would yet be $= \frac{1}{432}\,l$, as against $\frac{1}{360}\,l$.

The foregoing will prove, therefore, that the empirical rule given for depths of girders, in ordinary building practice, ought not to be objected to.